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# Who is this guy?

Born in 1972, raised in Marburg, Germany

MSc in Astrophysics (1996, U of London) Diploma in Physics (1998, U of Heidelberg) PhD in Astrophysics (2002, U of München)

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since 2003 research fellow in Uppsala (German and Swedish funding) since 2008 lecturer at Uppsala University

Research interests: stars from B to K, esp. at low(est) metallicity, chemical evolution of the Galaxy, quantitative spectroscopy, atomic diffusion, Gaia (coordinator for the computation of synthetic observables)

## What will be covered

I. Theoretical background (introductory)

observables, radiative transfer, opacities and line formation model atmosphere output how lines depend on  $T_{\text{eff}}$ , log g, log  $\varepsilon(X)$  etc.

II. Methods of stellar-parameter and chemicalabundance determination

fundamental stellar parameters photometry (in a nutshell) spectroscopy (a practical selection)

III. Exercise (tomorrow afternoon)

### Stellar spectroscopy in the early days



#### Stellar spectroscopy today



#### The Solar spectrum



#### What are stellar parameters?

There are different ways of looking at what defines a stars:

stellar-structure view  $M, \mathcal{L}, X, Y, Z, R, v_{rot}, t, ...$ 

stellar-atmosphere view  $\mathcal{F}_{v}, T_{\text{eff}}, \log g, [X_i/H], v_{\text{rot}} \sin i, ...$  $\log (GM/R^2)$ 

While the prior is (often) more fundamental, the latter is more directly related to observations (photospheres!) and generally speaking more applicable. In this lecture, I will follow the latter view.

## Linking input to output



**Observations** 

#### Precision vs. accuracy



NB: Some projects may require high precision *and* accuracy, while for others it will suffice to reach some level of precision.

## Stellar atmosphere: a definition

descriptive: the layers of a star from which we receive photons = the layers we can see

physical:

$$0 \leq \tau_v \leq 10$$

optical depth

where  $\tau_v = (-) \int_0^L \kappa_v \rho \, dx$  is the **optical height**, x measures the geometrical path [cm],  $\rho$  is the mass density [g cm<sup>-3</sup>],  $\kappa_v$  is the mass absorption coef  $\kappa_{v}$  is the mass absorption coefficient [cm<sup>2</sup> g<sup>-1</sup>] and L is the path length (see Gray, ch. 5, p. 113)

simple extinction law:  $\mathcal{I}(v) = \mathcal{I}_0(v) \exp(-\tau_v)$ 

#### Stellar atmospheres: typical figures

B

The Sun

Μ

$$M = 2 \times 10^{33} \text{ g} = \text{M}_{\odot}$$
$$R = 7 \times 10^{10} \text{ cm} = \text{R}_{\odot}$$
$$\mathcal{L} = 4 \times 10^{33} \text{ erg/s} = \text{L}_{\odot}$$

photosphere:  $\Delta R \approx 200 \text{ km} < 10^{-3} \text{ R}_{\odot}$   $n \approx 10^{15} \text{ cm}^{-3}$  $T \approx 6000 \text{ K}$ 

G



an O star  $M \sim 50 \text{ M}_{\odot}$   $R \sim 20 \text{ R}_{\odot}$  $\mathcal{L} \sim 10^6 \text{ L}_{\odot} (\propto M^3)$ 

photosphere:  $\Delta R \approx 0.1 \text{ R}_{\odot}$   $n \approx 10^{14} \text{ cm}^{-3}$  $T \approx 40\,000 \text{ K}$ 

O wikipedia: stellar classification

#### Abundance nomenclature

**Mass fractions**: let *X*, *Y*, *Z* denote the mass-weighted abundances of H, He and all other elements ("metals"), respectively, normalized to unity (X + Y + Z = 1). example: X = 0.739, Y = 0.249, Z = 0.012 for the Sun

**The 12 scale**:  $\log \epsilon(X) = \log (n_X / n_H) + 12 \ (\log \epsilon(H) = 12)$ 

example:  $\log \epsilon(O)_{\odot} \approx 8.7$  dex, i.e., oxygen, the most abundant metal, is 2000 times less abundant than H in the Sun (the exact value is currently hotly debated!)

**Square-bracket scale**:  $[X/H] = \log (n_X / n_H)_{\star} - \log (n_X / n_H)_{\odot}$ 

example:  $[Fe/H]_{HE0107-5240} = -5.3$  dex, i.e., this star has an iron abundance a factor of 200 000 below the Sun (Christlieb *et al.* 2002)

### Intensity and flux

The Sun is one of the few stars whose surface we can resolve ⇔ measure the so-called specific intensity

$$\mathcal{I}_{v} = dE_{v} / \cos \vartheta \, dA \, d\Omega \, dt \, dv$$
 [J / m<sup>2</sup> rad s Hz]

Usually, we measure stellar fluxes

$$\mathcal{F}_{v} = \int dE_{v} / dA dt dv \qquad [J / m^{2} s Hz]$$

Clearly, the flux  $\mathcal{F}_{v} = \int \mathcal{I}_{v} \cos \vartheta \, d\Omega$  and it measures the anisotropy of the radiation field.

Example: the Solar flux above the Earth's atmosphere  $\mathcal{F}(\odot) = 1.36 \text{ kW} / \text{m}^2$ 

# Flux constancy and luminosity

Stellar atmospheres are much too cool and tenuous to fuse nuclei

⇒ the energy coming from the stellar core is merely transported through the atmosphere, either by radiation or convection.

 $\mathcal{F}(x) = \text{energy} / \text{unit area} / \text{unit time}$  [J m<sup>-2</sup> s<sup>-1</sup>] = [W m<sup>-2</sup>]

$$d \mathcal{F}(x) / dx = 0 \qquad (generally: \nabla \mathcal{F} = 0)$$

The spectrum of  $\mathcal{F}$  (i.e.  $\mathcal{F}_{v}$ ) will change with *r*, but not the integral value.

If  $\mathcal{F}_{rad} \gg \mathcal{F}_{conv}$ , then one speaks of radiative equilibrium. (Karl Schwarzschild 1873–1916)

The total energy output of a star is called its **luminosity**  $\mathcal{L} = 4\pi R^2 \mathcal{F}(R)$ 

#### Stellar spectra

Luckily, **stars** (and other celestial bodies) are not in thermodynamic equilibrium (TE) and **do not shine like blackbodies**.



## **TE** statistics

Particle velocities are assumed to be Maxwellian:

$$\frac{n(v)}{n_{\text{tot}}} \mathrm{d}v = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}} \mathrm{d}v$$

Excitation follows the Boltzmann distribution:

$$\frac{n_u}{n_{\rm tot}} = \frac{g_u}{u(T)} e^{-\frac{\chi_u}{kT}}$$

Ionization can be computed via the Saha equation:

$$\frac{n_{\rm II}}{n_{\rm I}}P_e = \frac{(2\pi m_e)^{3/2}kT^{5/2}}{h^3} \frac{2u_{\rm II}(T)}{u_{\rm I}(T)}e^{-\frac{I}{kT}}$$

In **local thermodynamic equilibrium** (LTE), these are applied *locally*.







## The basics of radiative transfer

When the stellar photons interact with the stellar-atmosphere matter, photons can be absorbed and re-emitted. This is the basic message of the radiative transfer equation.

х

$$\begin{aligned} d\mathcal{I}_{v} &= -\kappa_{v} \rho \,\mathcal{I}_{v} \,dx + j_{v} \rho \,dx & \text{or} \quad j_{v} \text{: emission coefficient} \\ \cos \vartheta \, d\mathcal{I}_{v} \,/ \,d\tau_{v} &= +\mathcal{I}_{v} - \mathcal{S}_{v} & \text{with} \\ \mathcal{S}_{v} &= j_{v} / \kappa_{v} & \text{the source function} \\ \ln \text{LTE}, \, \mathcal{S}_{v} &= \mathcal{B}_{v} & \text{the Planck function} \end{aligned}$$

 $\mathcal{B}_{,,}$  has a number of wonderful properties: it does not depend on material properties (only T) and increases monotonically with increasing T for all v.

The integral  $\int \mathcal{B} \cos \vartheta \, d\Omega$  yields  $\sigma T^4$  (Stefan-Boltzmann law).

Similarly,  $T_{\text{eff}}$  is defined:  $\mathcal{F}_{Bol}(\text{Earth}) = \left(\frac{\theta}{2}\right)^2 \sigma T_{\text{eff}}^4$   $\theta$ : angular diameter bolometric flux above Earth's atmosphere

## Opacities

#### Continuous opacity

Caused by *bf* or *ff* transitions In the optical and near-IR of cool stars, H<sup>-</sup> (I = 0.75 eV) dominates:  $\kappa_v(\text{H}^-_{bf}) = \text{const.} T^{-5/2} P_e \exp(0.75/kT)$ 

NB: There is only 1 H<sup>-</sup> per 10<sup>8</sup> H atoms in the Solar photosphere.

Line opacity (all the lines you see!) Caused by bb transitions Need to know loggf, damping and assume an abundance



## Model atmosphere output

A 1D model atmosphere is a tabulation of various quantities as a function of (optical) depth:

T (temperature)

 $P_{g}$  (gas pressure)

 $P_{\rm e}$  (electron pressure)

 $\mathcal{F}_{v}$  (esp. *surface* flux) etc. as computed under certain input assumptions:

 $T_{\text{eff}}$  (effective temperature) log g (surface gravity) log  $\varepsilon(X_i)$  (chemical composition) hydrostatic equilibrium

LTE (local thermodynamic equilibrium) MLT (mixing-length theory) and a statistical representation of opacities (either via opacity distribution functions, ODF, or opacity sampling, OS).



#### How spectral lines originate



## Spectral lines as a function of abundance

Starting from low log  $\varepsilon$  (low log gf), the line strength is directly proportional to log gf $\varepsilon$ :

$$W_\lambda \propto gf n_X$$

When the line centre becomes optically thick, the line begins to saturate. The dependence on abundance lessens. Only when damping wings develop, the line can grow again in a more rapid fashion:

 $W_{\lambda} \propto \operatorname{sqrt}(\operatorname{gf} n_{\mathrm{X}})$ 

Weak lines are thus best suited to derive the elemental composition of a star, given that they are well-observed (blending!)



# Broadening of spectral lines

There are numerous broadening mechanisms which influence the strength and apparent shape of spectral lines:

- 1. natural broadening (reflecting  $\Delta E \Delta t \ge h/2\pi$ ) 2. thermal broadening **3. microturbulence**  $\xi_{micro}$ (treated like extra thermal br.) (4. isotopic shift, hfs, Zeeman effect) **5.** collisions (H:  $\gamma_6$ , log C<sub>6</sub>; e<sup>-</sup>:  $\gamma_4$ ) (important for strong lines) 6. macroturbulence  $\Xi_{\rm rt}$ 7. rotation
  - (8. instrumental broadening)



Fig. 3. Synthetic (half-)profiles of Fe I 6151.6 Å (Mult. 62, E.P. = 2.2 eV) showing the cumulative effect of various broadening mechanisms.

macro

#### Microturbulence and damping

- If lines of intermediate or high strength return too high abundances, then the microturbulence or the damping constants are (both) underestimated (the gf values can also be systematically off).
- **Use an element with lines of all strengths to determine** ξ. In most cases, this will be an irongroup elements.
- Hydrodynamic ("3D") models are presently in an adolescent phase and will hopefully do away with the need for micro/macroturbulence.







Fig. 5. Iron abundances derived from individual solar Fe I lines and Hannover gf-values. The two samples shown are from [4] (squares) and [18] (triangles). The deviation of the stronger lines indicates that the adopted damping constants are too small.

#### Broadening of spectral lines: an example

The Ca II triplet lines are broadened by elastic collisions with hydrogen:

 $Ca + H \rightarrow Ca^* + H^*$ 

Detuning 
$$\Delta v = C_n / R^n$$
: here  $C_6$ 

Progress in the QM description of this interaction has led to a better understand of the profiles of these (and many other) lines (Anstee & O'Mara 1991, 1995).



## Spectral lines as a function of $T_{eff}$

The strength of a weak line is proportional to the ratio of line to continuous absorption coefficients,  $l_v / \kappa_v$ . Evaluation of this ratio can tell us about the  $T_{\text{eff}}$  sensitivity of spectral lines:

R =  $l_v / \kappa_v$  = const.  $T^{5/2} / P_e \exp((\chi + 0.75)/kT)$ for a neutral line of an element that is mostly ionized.

Fractional change with T: 1/R dR/dT =  $(\chi + 0.75 - I)/kT^2$   $\Rightarrow$  depending  $\chi$  on **neutral lines decrease with**  $T_{\text{eff}}$  by between 10 and 30% per 100 K (typically 0.07 dex per 100 K). Lines of different  $\chi$ can be used to constrain  $T_{\text{eff}}$  (**excitation equilibrium** condition).

For **ionized lines** of mainly ionized elements, one finds low sensitivities to  $T_{\rm eff}$ , except those **with a large**  $\chi$ . These **become stronger with**  $T_{\rm eff}$  by up to 20% per 100 K.

# Spectral lines as a function of $\log g$

The  $T_{\rm eff}$  sensitivity of spectral lines may be surpassed by sensitivities with respect to other stellar parameters.

#### Sensitivity to log g in cool stars?

Case 1: (weak) neutral line of an element that is mainly ionized  $W_{\lambda}$  is proportional to the ratio of line to continuous absorption coefficients,  $l_{\nu} / \kappa_{\nu}$ .  $n_{r+1} / n_r = \Phi(T) / P_e \quad \Leftrightarrow \quad n_r \approx \text{const. } P_e$  $\Rightarrow l_{\nu} / \kappa_{\nu} \neq f(P_e) \quad \text{neutral lines do not depend on } \log g$ 

Case 2: ionized line of an element that is mainly ionized (**universal**) log *g* sensitivity via the continuous opacity of H<sup>-</sup>

NB: for strong lines, a damping-related log g sensitivity comes into play.

# LTE vs. NLTE

Occupation, excitation & ionization are assumed to be local properties ⇒ Saha-Boltzmann statistics

Assuming the T-P- $\tau$  relation to be known, all you need to to calculate a line strength is

- (a) the level energies and statistical weights involved
- (b) the transition probability
- (c) broadening mechanisms (microturbulence, van-der-Waals damping)

Photons carry non-local information

Occupation, excitation & ionization depend on the microphysics (radiation field, collisions etc.)

One needs to know (and master!) a whole lot of atomic physics.

One also needs to solve the involved numerical problem of radiative transfer plus **rate equations**:

 $n_{\rm i} \sum_{\rm j \neq i} \left( R_{\rm ij} + C_{\rm ij} \right) = \sum_{\rm j \neq i} n_{\rm j} \left( R_{\rm ji} + C_{\rm ji} \right)$ 

While LTE may be an acceptable approximation for a cool-star photosphere on the whole, it can be very wrong for specific lines.

### Fundamental stellar parameters

 $T_{\text{eff}}$ : via  $\mathcal{F}_{\text{Bol}}$  and  $\theta$  (see IRFM below). To get  $\theta$ , one uses interferometry and model-atmosphere theory (limb darkening!).

log g: Newton's law, needs M and R. So usually one needs  $\pi$  (parallax) and  $\theta$ . Gaia is the key  $\pi$  mission (launch 2012).

*M* needs to be inferred from stellar evolution.

Exception: eclipsing binaries.

[*m*/H]: via meteorites (only for the Sun), which lack important (volatile) elements like CNO and noble gases. In principle, asteroseismology can provide compositions of other stars.



#### Photometry: pros vs. cons

Photometry is

- an efficient way of determining stellar parameters,
- $\checkmark$  can probe very deep,
- ✓ freely available (surveys!),
- $\checkmark$  comparatively cheap to obtain.

However, photometry is

- limited in which parameters can be derived,
- subject to extra parameters (reddening!)
- subject to parameters that cannot be determined well (ξ, [α/Fe]).



#### Photometric standard systems



Warning: there is often more than one filter set for one system!

## Photometry: $T_{\text{eff}}$ dependence

T<sub>eff</sub> variations dominate the flux variations of cool stars.
 In the BB approximation to stellar fluxes, it suffices to measure the flux at two points to uniquely determine *T*. In reality, [*m*/Fe] and reddening complicate the derivation of photometric stellar parameters.



### Photometry: metallicities

- After  $T_{eff}$ , the global metallicity has the largest influence on stellar fluxes (with the potentially disastrous exception of reddening!).
- But the **precision** with which metallicities can be determined **is limited** (of order 0.3 dex). In addition, it is difficult to determine metallicities for stars with [Fe/H] < -2, as classical indicators like  $\delta(U - B)$  lose sensitivity.
- On the other hand, there are narrow-band indices which allow one to measure abundance variations (e.g. via molecular bands).



### Photometry: gravity dependence

The only feature that has a sufficiently (?) large gravity sensitivity to be exploited by photometry is the **Balmer jump at 3647** Å (in hot stars it can be used as a sensitive  $T_{\text{eff}}$  indicator).

Colours like (U - B) or (u - y)measure the Balmer discontinuity, but the usefulness as a precise gravity indicator is hampered by the high line density in this spectral region (missing opacity problem), the difficulties with ground-based observations in the near-UV and a proper treatment of the overlapping Balmer lines.



The c<sub>1</sub> index (= (u - b) - (b - y)) works well for metal-poor giants (Önehag *et al.* 2008).

# IRFM: a semi-fundamental $T_{\text{eff}}$ scale

Basic idea of the infrared-flux method:

$$\frac{\mathcal{F}_{\rm I} \text{ (surface)}}{\mathcal{F}_{\lambda_{\rm IR}} ({\rm Earth})} = \frac{\sigma T_{\rm eff}^4}{\mathcal{F}_{\lambda_{\rm IR}} ({\rm model})}$$

- $\mathcal{F}_{\lambda IR}$ (model) is said to be only weakly model dependent (but cf. Grupp 2004).
- Once calibrated on stars with known diameters, any colour index can be calibrated on the IRFM.

**Direct** sample:  $\Delta T_{\text{eff}} = 0.06 \pm 1.25\%$ 

Comparing different IRFM calibrations (Blackwell *et al.*, Ramírez & Meléndez, Casagrande *et al.*), the **zero point** proves to be **uncertain by ±100 K**, in particular for metal-poor stars.



# Spectroscopic $T_{\text{eff}}$ indicators: H lines

- Above 5000 K, the wings of Balmer lines are a sensitive  $T_{eff}$  indicator, broadened by H + H collisions (mainly H $\alpha$ ) and the linear Stark effect (H + e<sup>-</sup>).
- In cool stars, the log g sensitivity is low (line and continuous opacity both depend on  $P_e$ ), as is the metallicity dependence. There is some dependence on the mixing-length parameter (H $\beta$  and higher).
- Main **challenge** (apart from the surprisingly complex broadening): **recovering the intrinsic line profiles** from (echelle) observations.
- In hot stars, Balmer lines can constrain the surface gravity.



## H $\alpha$ as a function of $T_{\rm eff}$



## Line-depth ratios (LDRs)

Using the ratio of two lines' central depths (rather than  $W_{\lambda}$ ) can be a remarkably sensitive temperature indicator (precision as high as 5 K!), if the lines are chosen to have different sensitivities to *T*. Ideally, the LDR is close to 1 and the lines should not be too far apart.



Gray Fig.14.7

The main challenge lies in a proper  $T_{\text{eff}}$  calibration across a usefully large part of the HRD.

## Gravity sensitivity of ionized lines

Recall that ionized lines of an element that is mainly ionized have a  $P_e^{-1}$  sensitivity via the continuous opacity of H<sup>-</sup>.

#### Hydrostatic equilibrium

$$dP/d\tau_v = g/\kappa_v$$



## Practicalities of ionization equilibria

#### A change of **0.1 dex in log ε** translates to a change of **0.3 dex in log g**.

Consequences:

A line-to-line scatter of 0.1 dex means that  $\log g$  is known to within 0.3 dex.

Relatively small changes in log  $\varepsilon$ , e.g. because of a change in  $T_{\rm eff}$ or NLTE effects, can lead to factor-of-two changes in the surface gravity.

Astrometry can help to establish the correct surface-gravity scale.



Korn (2004), Carnegie Observatories Centenary (2003) http://www.ociw.edu/ociw/symposia/series/symposium4/proceedings.html

#### The strong line method

Damped (neutral) lines show a strong gravity sensitivity, because

 $l_{
m v} \propto \gamma_6 \propto P_{
m g} \propto g^{2/3}.$ 

Like with ionization equilibria, log  $\varepsilon$  needs to be known. This is to be obtained from weak lines of the same ionization stage, preferably originating from the same lower state (no differential NLTE effects).



Examples: Ca I 6162 (see above), Fe I 4383, Mg I 5183, Ca I 4226. Below [Fe/H]  $\approx$  -2, there are no optical lines strong enough to serve as a surface-gravity indicator.

## Spectroscopy of the Solar neighbourhood

#### Aim:

Derive precise stellar parameters and chemical abundances of FGK stars within d = 25 pc.

#### Example:

- The strong-line method as a surface-gravity indicator for not too metal-poor, not-tooevolved stars
  - coupled with  $T_{\rm eff}$  values from Balmer lines.

#### **Benchmark: Hipparcos**



# Abundances from H to U

Once you have good stellar parameters, it is relatively easy to determine chemical abundances for your favourite element(s).

#### Caveats

- some elements are not visible, e.g. noble gases in cool stars
- lines may lack or have inaccurate atomic data
- lines can be blended leading to overestimated abundances
- lines can be subject to effect you are unaware of, e.g. 3D and NLTE effects, hfs, isotopic and Zeeman splitting



#### Quantitative spectroscopy: the Sun



#### Spectroscopy: pros vs. cons

#### Spectroscopy is

- a way of determining a great number of stellar parameters,
- ✓ the key technique for obtaining detailed chemical abundances,
- ✓ (usually) reddening-free.

However, hi-res spectroscopy is

- comparatively costly at the telescope,
- $\Box$  currently limited to  $18^{m}$  in *V*,
- more difficult to master than photometry.



...especially when they accept photometry as a source of valuable information.

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