Transiting exoplanets
Characterization of hot Jupiters

Carolina von Essen

Stellar Astrophysics Centre, Aarhus University
Moletai Observatory, August 2016
Today

- I) Transiting exoplanets, their fundamental parameters
- II) Limb-darkening coefficients
- III) Fitting a transit: Markov Chain Monte Carlo (MCMC)
- IV) Exoplanet atmospheres
Part I
Transiting exoplanets

- Up to 3000 exoplanets discovered. How many of those transit? 
  http://exoplanet.eu/
- Some of them discovered from the ground... WASP-10? 
  http://var2.astro.cz/ETD/
- While others from space. Kepler-36? 
  http://exoplanetarchive.ipac.caltech.edu/
How do transits look like?

Transit depth and transit duration!! Which are the transiting parameters?
Shape of transits

- Change inclination: 90, 86, 80.
- Change semi-major axis: 6.5, 7.5, 8.5.
- Change orbital period: 2, 2.5, 3.5.
- Change T0: 0.2, 0.3, 0.35.
- Change Rp/Rs: 0.1, 0.12, 0.14.

Follow program_part1.py
Part II
Limb-darkening coefficients

The Sun: any features calling your attention? Try $\text{linLimb} = 0.7$ and $\text{quadLimb} = 0.14$. What changes?
Limb-darkening coefficients

- Limb-darkening laws describe how does the stellar intensity changes from the center to the limb.

- How the intensity changes depends on the star. Name 3 most fundamental stellar parameters.

- We adjust a “limb-darkening law” to the change of intensity of the star...

\[
\frac{I(\mu)}{I(\mu = 1)} = 1 - u(1 - \mu) \quad \text{Linear, (1)}
\]

\[
\frac{I(\mu)}{I(\mu = 1)} = 1 - a(1 - \mu) - b(1 - \mu)^2 \quad \text{Quadratic, (2)}
\]

\[
\frac{I(\mu)}{I(\mu = 1)} = 1 - c(1 - \mu) - d(1 - \sqrt{\mu}) \quad \text{Square-Root, (3)}
\]

\[
\frac{I(\mu)}{I(\mu = 1)} = 1 - \sum_{j=1}^{4} f_j (1 - \mu^{j/2}) \quad \text{4-Parameter, (4)}
\]

\[
\frac{I(\mu)}{I(\mu = 1)} = 1 - g(1 - \mu) - h \frac{1}{1 - e^{\mu}} \quad \text{Exponential, (5)}
\]

\[
\frac{I(\mu)}{I(\mu = 1)} = 1 - m(1 - \mu) - n\mu \ln \mu \quad \text{Logarithmic, (6)}
\]

... and we get the limb-darkening coefficients!
Limb-darkening coefficients

Depend on the star, and on the wavelength the star is observed!
Limb-darkening coefficients

You can get them from others...

- Choose your limb-darkening law.
- Choose your stellar parameters (mostly $T_{eff}$, log(g) and [Fe/H]).
- Choose your filter (examples: B, V, R, I). **What is a filter and a transmission function?**
- Go to vizier: http://vizier.u-strasbg.fr/
- phoenixc option.
- Write down the quadratic limb-darkening coefficients.
Limb-darkening coefficients

Or you can make them yourself!

- Go to: http://phoenix.astro.physik.uni-goettingen.de/
- Choose as close as possible stellar parameters.
- Download fits file.
- Google filter transmission function.
- Digitalize it with engauge (make sure first and last values are 0).
- **Follow the program_part2.py program.**
Part III
Bayesian Statistics (BS)

- Real phenomena can be modelled by parameters, $\theta$. Example $\theta$: the medium height of a population.
- BS treats the parameters as unknown and aleatory.
- But!!!! The goody: we can have an idea of what these parameters should be, and consider this in our analysis.

CAREFUL!! This is dangerous. Its important to properly reflect our knowledge (and ignorance too!!) of the parameters.

Example on transits?
Bayesian Statistics (BS)

In general:

\[ \text{Posterior} \propto \text{Prior} \times \text{Likelihood} \]

\[ P(\theta|\text{Data}) = \frac{P(\theta)P(\text{Data}|\theta)}{P(\text{Data})} \]

- \( P(\theta) \): **Prior distribution.** Prior knowledge of the values that \( \theta \) can take.
  Example: normal or uniform distributions.

- \( P(\text{Data}|\theta) \): **conditional probability (likelihood).** It indicates the compatibility between the parameters and the data \( \rightarrow \chi^2 \! \)

- \( P(\theta|\text{Data}) \): **posterior distribution.** is the probability of \( \theta \) given the data.

BLA BLA ZZZzzZZzzzzZZzzZZzz.
The un-boring concept

Posterior $\propto$ Prior x Likelihood

- Parameter: orbital inclination, $i$.
- Prior: $i \sim 90$. But can I constrain the $i$ more? Prior!!
- I use MCMC (its coming :-) to evaluate $i$ in the model, and I ask myself: Does the model($i$) looks like the observations?? Likelihood!!
- If it looks like (ex. $\chi^2$) “I keep it”.
- What do I get at the end? $i \pm \delta i$. Posterior!!!
Careful with the priors

If you don’t know so well about the parameters → reflect that.
If you do know → reflect that too!
I want $a \pm \delta a$, $i \pm \delta i$, $P \pm \delta P$. So I fit a transit.

![Graph](image)

Figure 4. 84 individual *Kepler* light curves were used to create this phase-folded and binned light curve of TrES-2. Always 125 measurements were combined into one data point.

How do I construct the priors on $i$?
Careful with posteriors!

What does $i \pm \delta i$ mean? Let's check the posteriors out.

$\mu, \sigma$: quantities from the posterior distributions.
When do I use MCMC?
1) “Simple” example: linear regression analysis.

We want to find $a, b$ that best fit the data.
We can minimize the error

$$E(a, b) = \sum_{i=1}^{N} [y_i - (a + bx_i)]^2$$

$$\frac{\partial E(a,b)}{a} = 0, \frac{\partial E(a,b)}{b} = 0.$$

This has analytic solution for $a, \delta a, b$ and $\delta b$. But is a linear fit!! For more parameters and complicated model $\rightarrow$ MCMC!!
MC + MC

Monte Carlo:

- involves to simulate N independent and random values (example: people).
- N grows \( \rightarrow \) approximation improves (law of large numbers).

Markov Chain:

- Converges (not always) after N\( \gg \) to the desired probability distribution.
- Depends only on current step.
- Evolves randomly till equilibrium.
MCMC

But if I am “wondering like a crazy person”, how do I reach equilibrium?

**Random walk**: “Jumps” that are accepted or not, according to a given probability ($\chi^2$!!!)

*Statistics is art.* Convergence depends on step!!!
Nice chain... bad chain.

How do I analyze if the chain converged?
MCMC chains: the “art” part

- Limits: uniform or normal?
- Step: what size?
- How many iterations?
- What about a burn-in?
- Thin?
- $\mu, \sigma$.
- Correlation between parameters?
- Did my chains converge?

First step to analyze MCMC chains:

PyAstronomy :-) $\rightarrow$ program_part3.py
Exoplanet atmospheres

Exoplanet spectroscopy: why is it interesting?

- Chemical composition
  - Interior composition
  - Formation and evolution
- Clouds, hazes.
- Rayleigh dispersion.
- Physical profiles such as T-Pr.
- Grouping in categories for comparative studies.
- The discovery of an Earth-like planet in the habitable zone.
Intuitive approach: direct imaging

Big obstacles:

► Huge contrast between the planet and star. From $1:10^3$ for a hot-Jupiter in the NIR, to $<1:10^9$ for an Earth-like planet (solar star, 1 AU).

► Tiny angular separation between planet and star. However, direct imaging, succeed!

Direct imaging: outstanding results

Marois et al. (2010)

GPI first-light images of HR4796A.
Exoplanet atmospheres

Indirect methods using transiting exoplanets.

1. Primary transit.
2. Secondary transit.
3. Phase curve.

Winn (2011).
Exoplanet atmospheres

Albedos and temperatures from photometry!
Transmission spectroscopy

- Aims at characterizing the atmosphere from analyzing variations in spheric ring with thickness $\delta$.

$$\delta = \frac{R_P}{R_S^2} H$$

- Scale height

$$H = \frac{kT}{\mu g}$$
Transmission spectroscopy

- For an averaged hot-Jupiter transiting a solar-like star \( \delta \sim 10^{-4} \).
- For an Earth-like planet transiting a solar-like star \( \delta \sim 10^{-6} \).
- Targets we can tackle: \( \delta \sim 10^{-3} \).
Different atmospheric models
Different atmospheric models

Techniques: high resolution

- Longitud de Onda (nm)
- Absorción producida por la estrella
- Exceso en absorción debido al planeta
- Estrella tipo solar
- Luz absorbida por el planeta
- Exceso de luz absorbida por el planeta
- Duración del tránsito
- Brillo de la estrella
Techniques: low resolution

- Observation of target+reference simultaneously.
- Instruments mostly used: HST, GEMINI/GMOS.
- "Binning" of the spectra in channels.
- Build-up of light curves as a function of color.
- Determination of $R_P/R_S$. 
Techniques: low resolution

GJ-1214 b, HST/WFC3, Kreidberg et al., (2014)
Techniques: low resolution

HD-189733 b, Sing et al., (2011)
Solid coloured lines show fitted atmospheric models with prominent spectral features indicated. Planets with predominantly clear atmospheres (top) show prominent alkali and H2O absorption. Very hazy and cloudy planets (bottom) have strong optical scattering slopes, narrow alkali lines and H2O absorption that is partially or completely obscured.

Sing et al. (2016)
What we will do in the upcoming days

- Day 1) Photometric data reduction of transit. IRAF.
- Day 2) Transit fitting, MCMC. Python.
- Day 3) Spectroscopic data reduction of transit. IRAF.
- Day 4) Construction of the color-dependent light curves. Determination of $R_P/R_S(\lambda)$. 