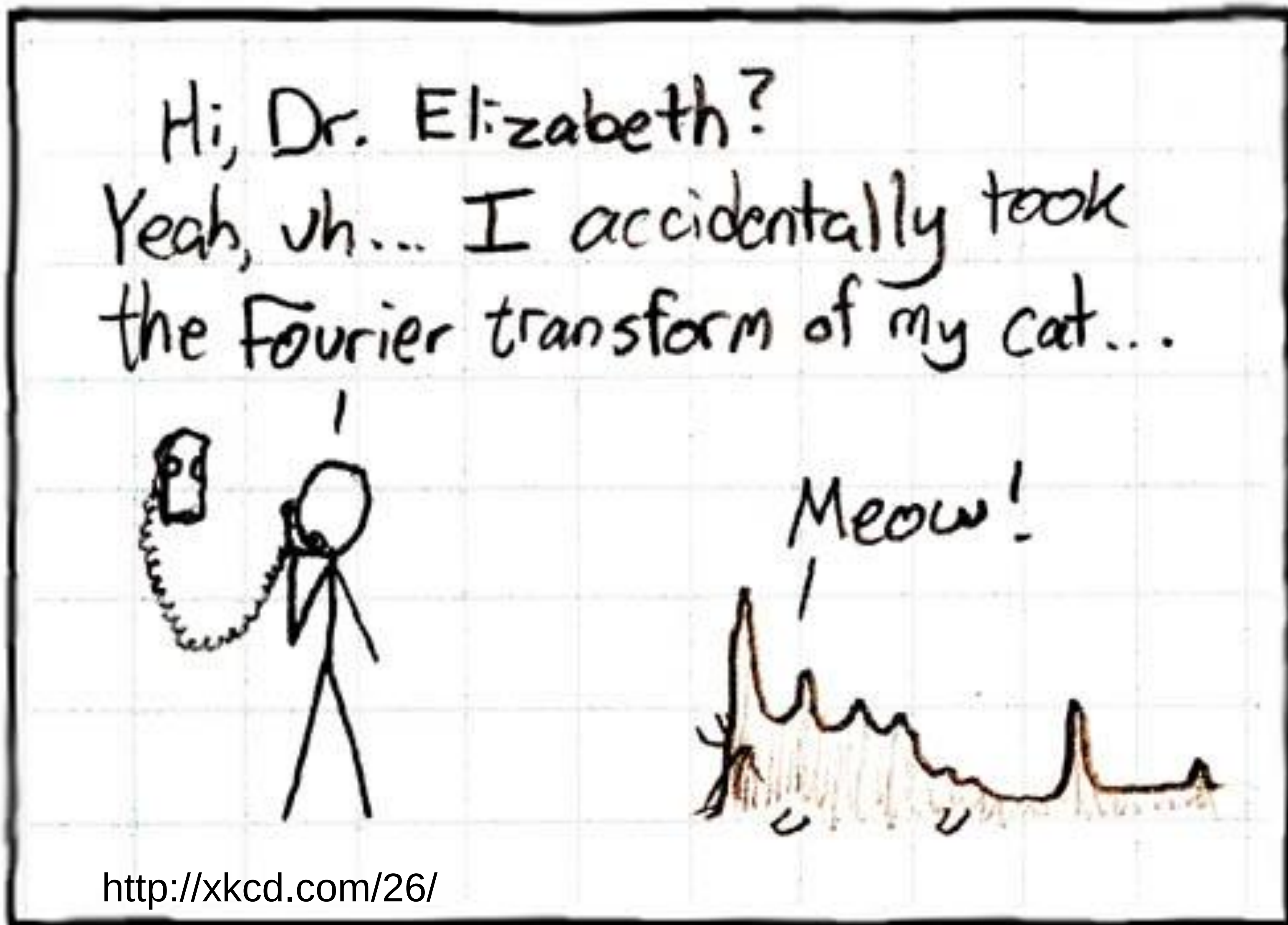
Some tips on Fourier Analysis



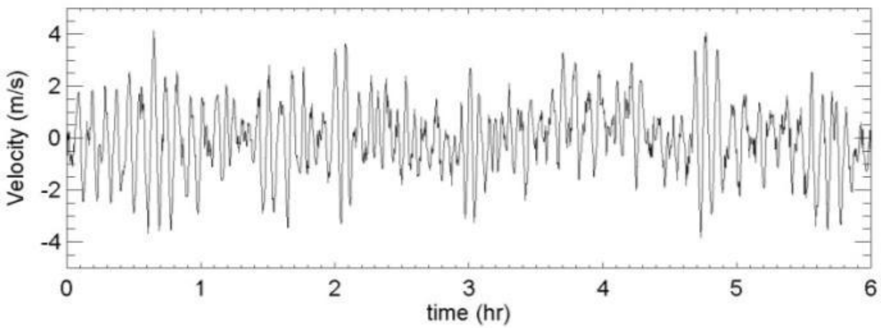
http://xkcd.com/26/

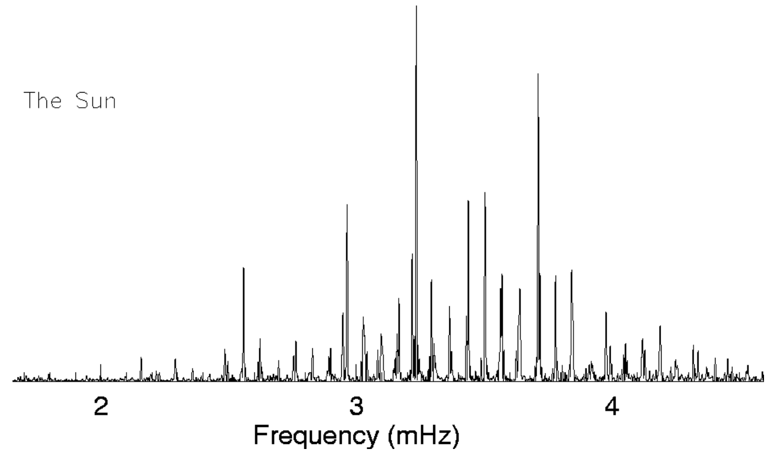
Asteroseismology:

- the study of “starquakes”
- probing stellar interiors using their oscillations

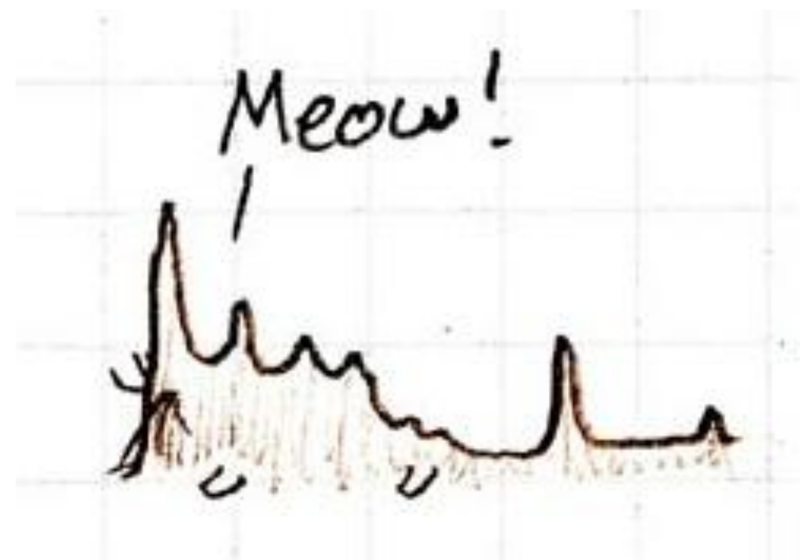
Topics

- What are stellar oscillations?
- Why do stars oscillate?
- How do we observe these oscillations?
- How do we analyse the data?
- What are we learning?



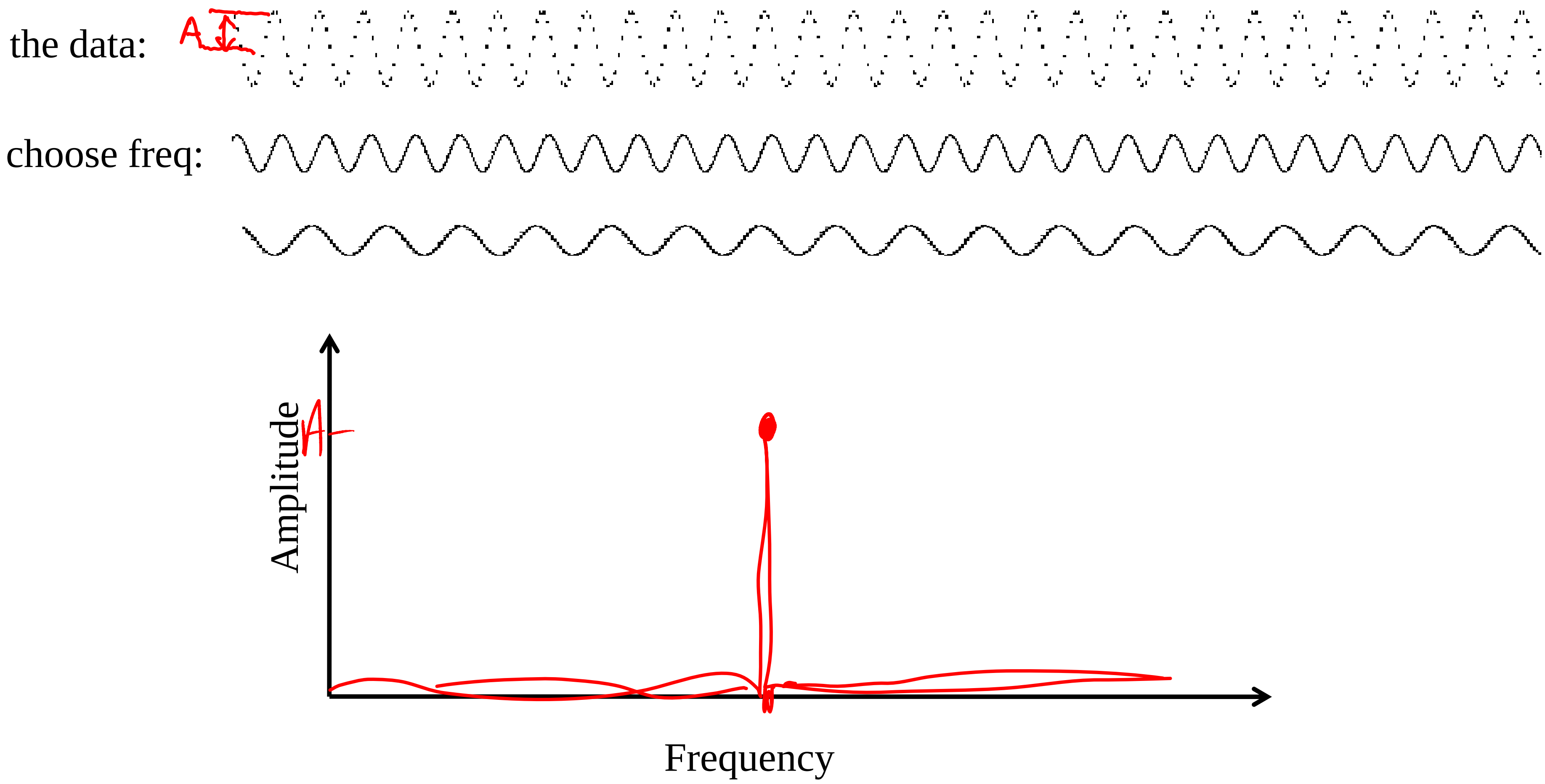


- amplitude spectrum tells you which frequencies are present in the time series (a Fourier-transform relationship)
- power is just |amplitude|²

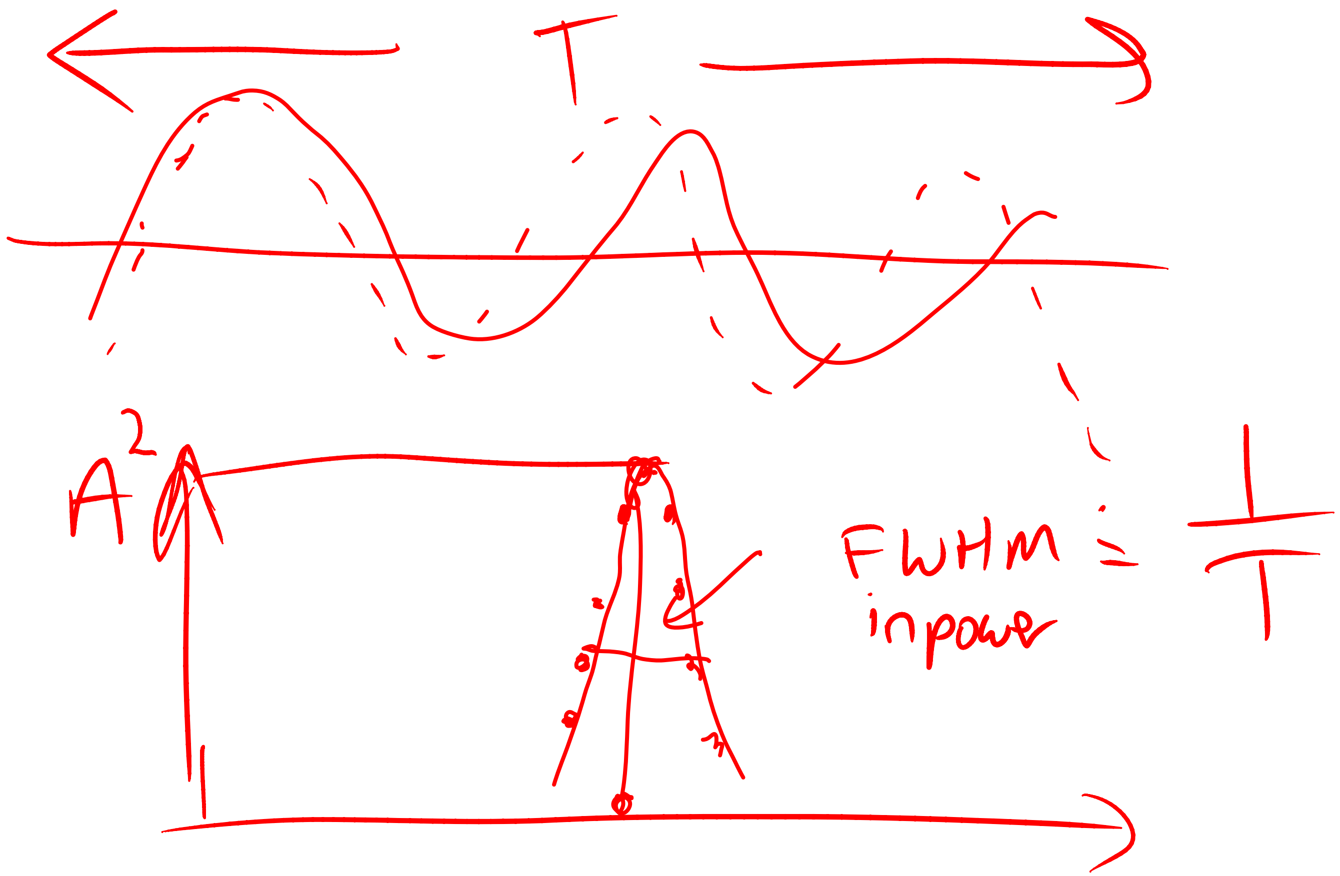


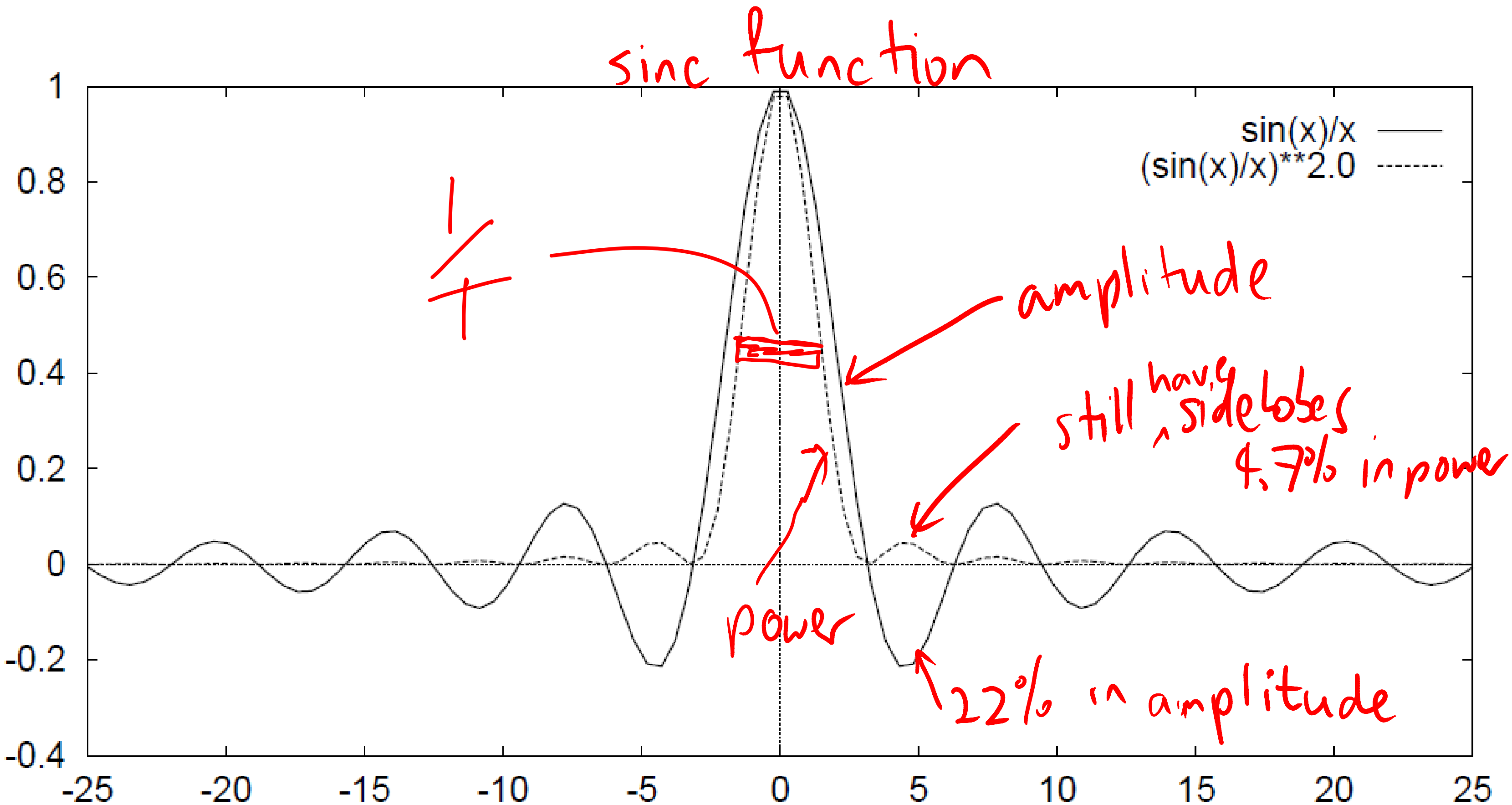
The Recipe:

Choose a frequency. Do a least-squares fit of a sine wave (varying the amplitude and phase) and plot the best-fitting amplitude.



what if we pick slightly the wrong frequency?

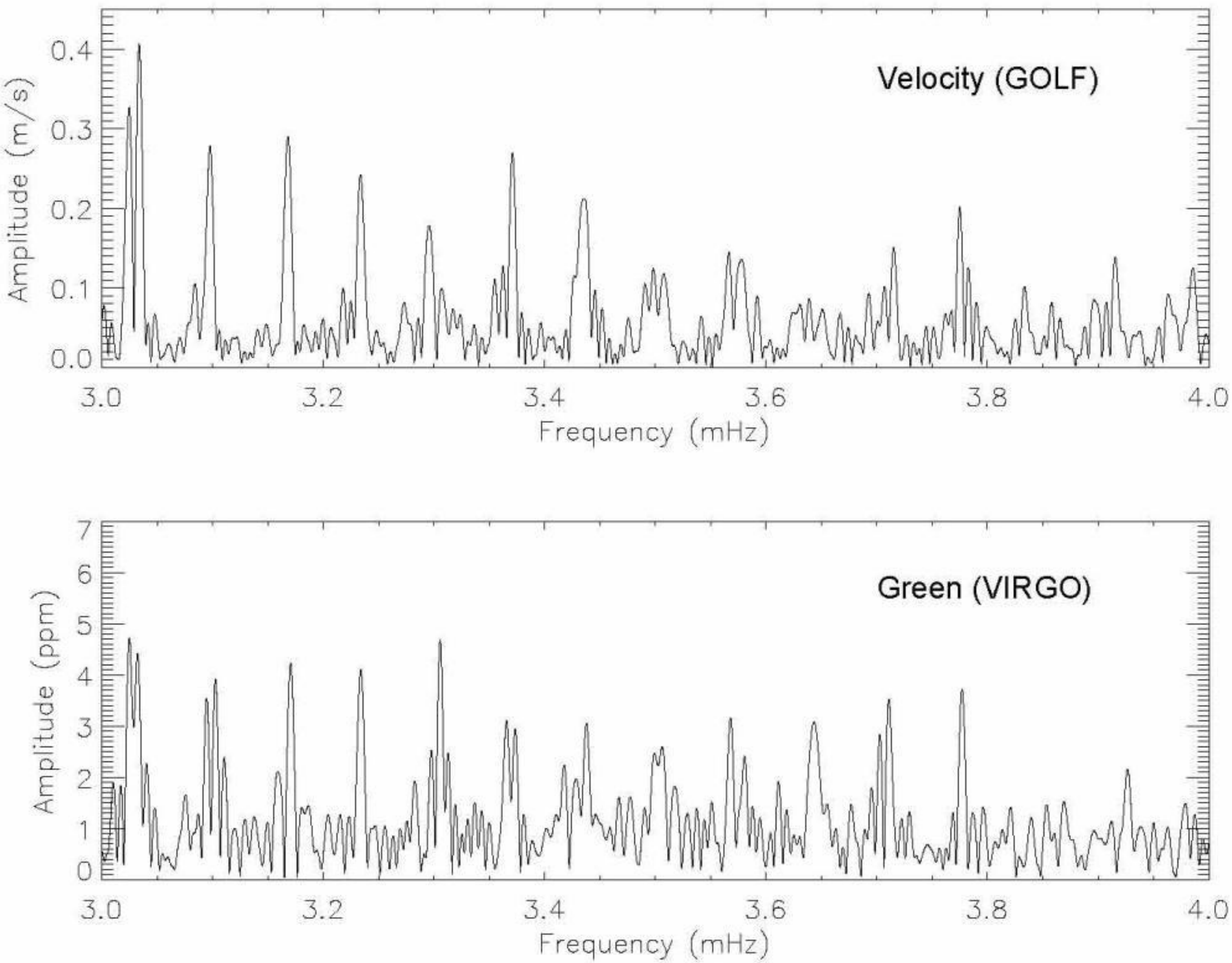




FWHM in power is 1/(total observing time)

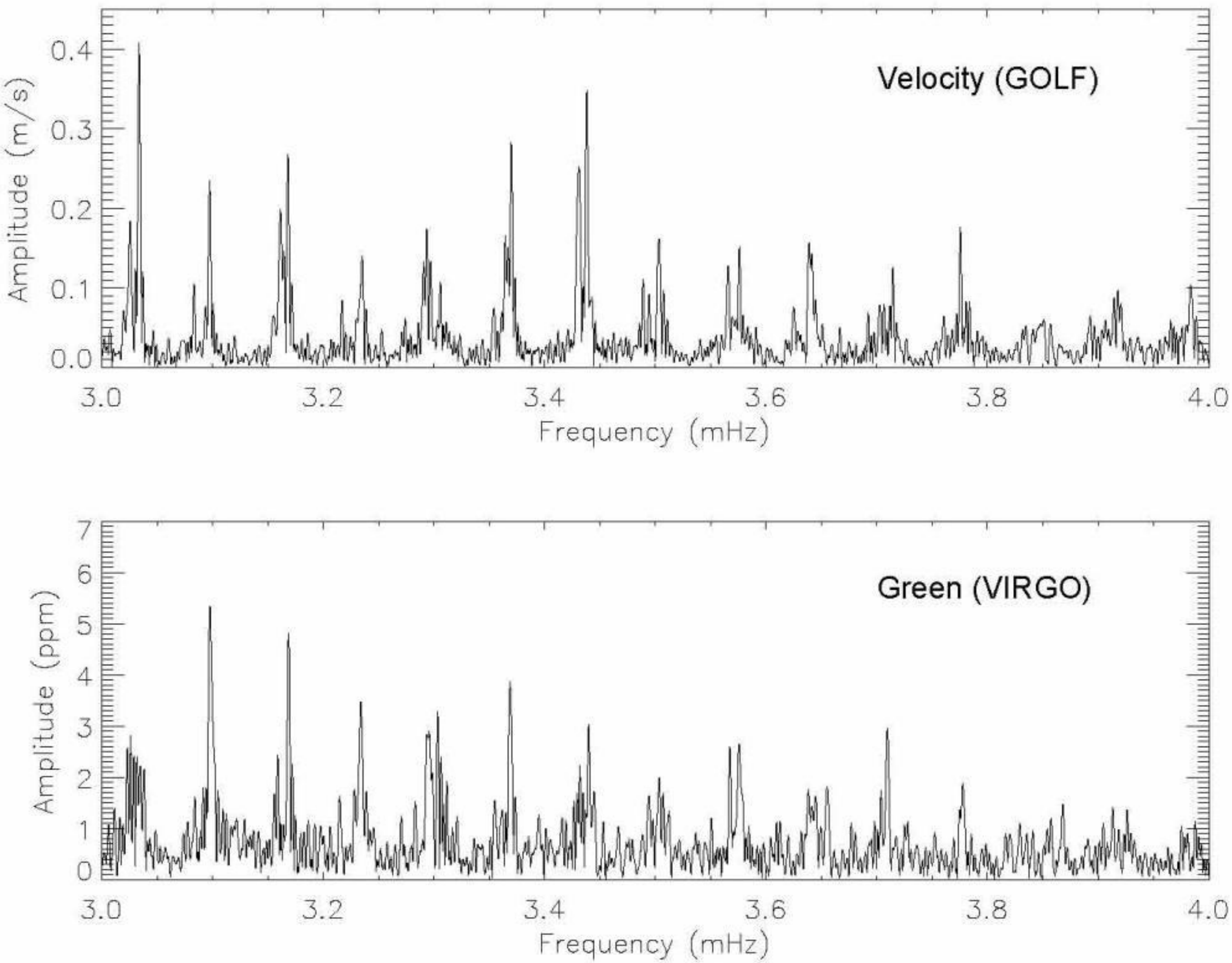
what happens as time series gets longer?

2 days



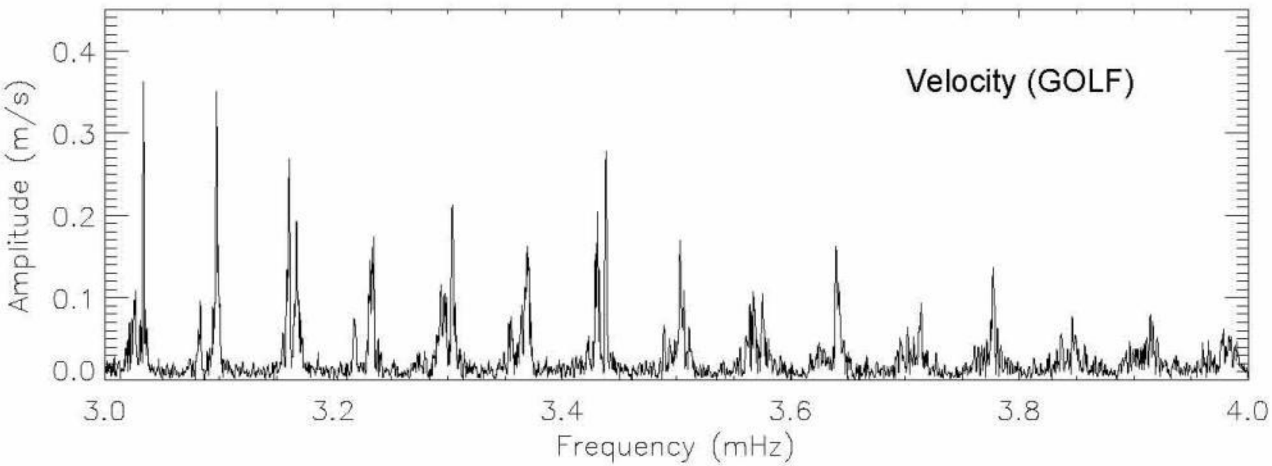
Velocity (GOLF)
Amplitude (m/s)
0.0
0.1
0.2
0.3
0.4
3.0
3.2
3.4
3.6
3.8
4.0
Frequency (mHz)
Green (VIRGO)
Amplitude (ppm)
0
1
2
3
4
5
6
7
3.0
3.2
3.4
3.6
3.8
4.0
Frequency (mHz)

5 days

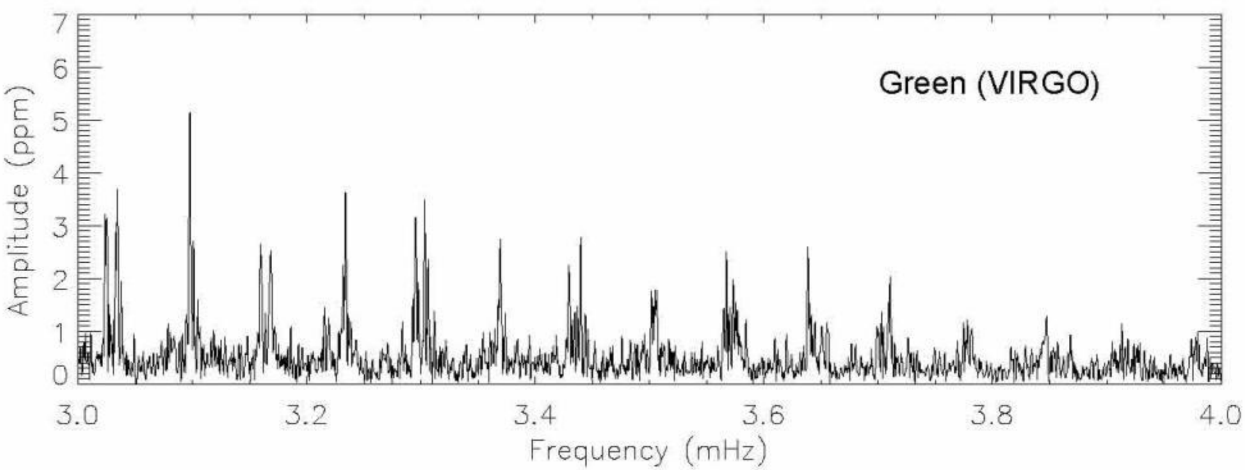


Velocity (GOLF)
Amplitude (m/s)
0.0
0.1
0.2
0.3
0.4
3.0
3.2
3.4
3.6
3.8
4.0
Frequency (mHz)
Green (VIRGO)
Amplitude (ppm)
0
1
2
3
4
5
6
7
3.0
3.2
3.4
3.6
3.8
4.0
Frequency (mHz)

10 days

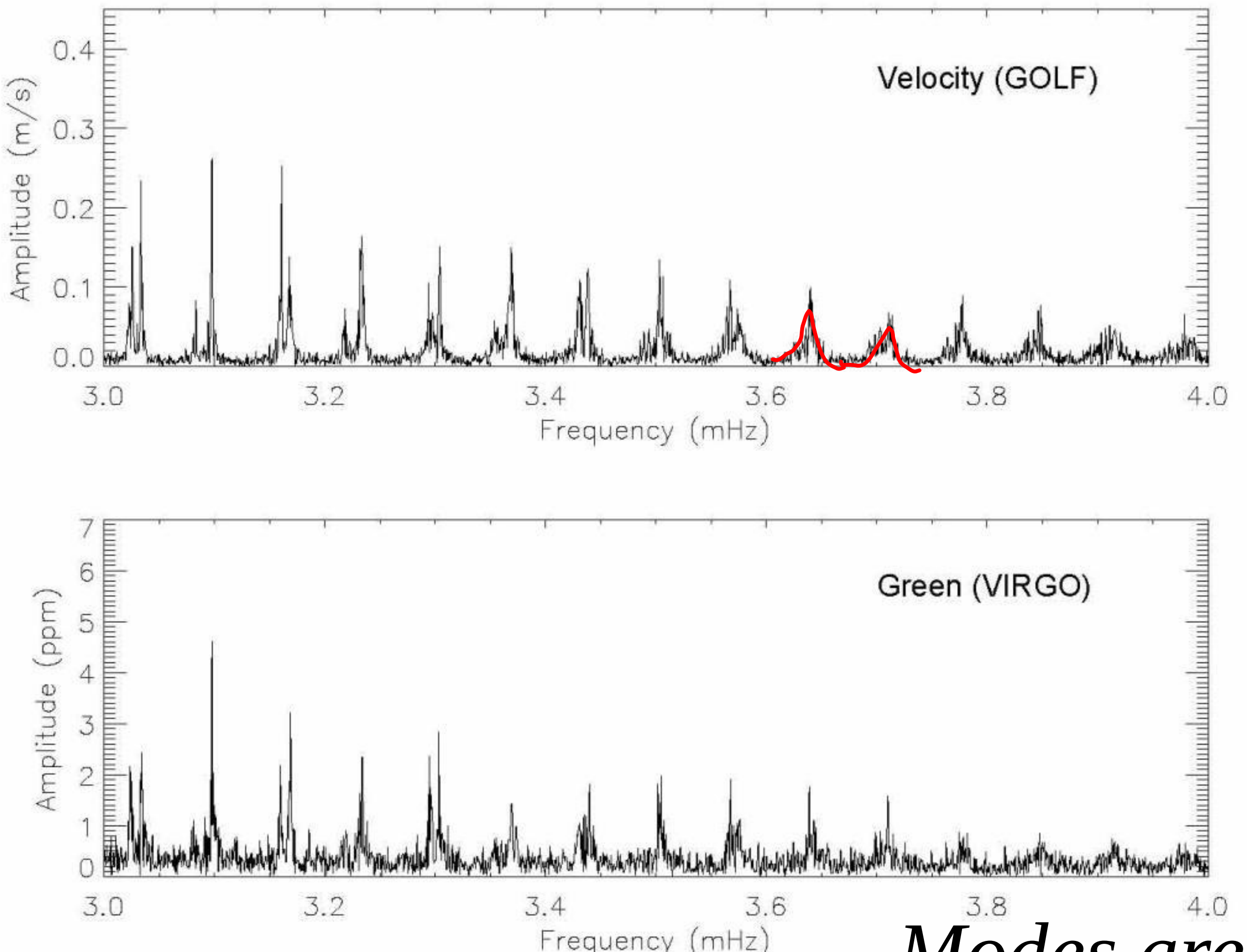


Velocity (GOLF)
Amplitude (m/s)
0.4
0.3
0.2
0.1
0.0
3.0
3.2
3.4
3.6
3.8
4.0
Frequency (mHz)



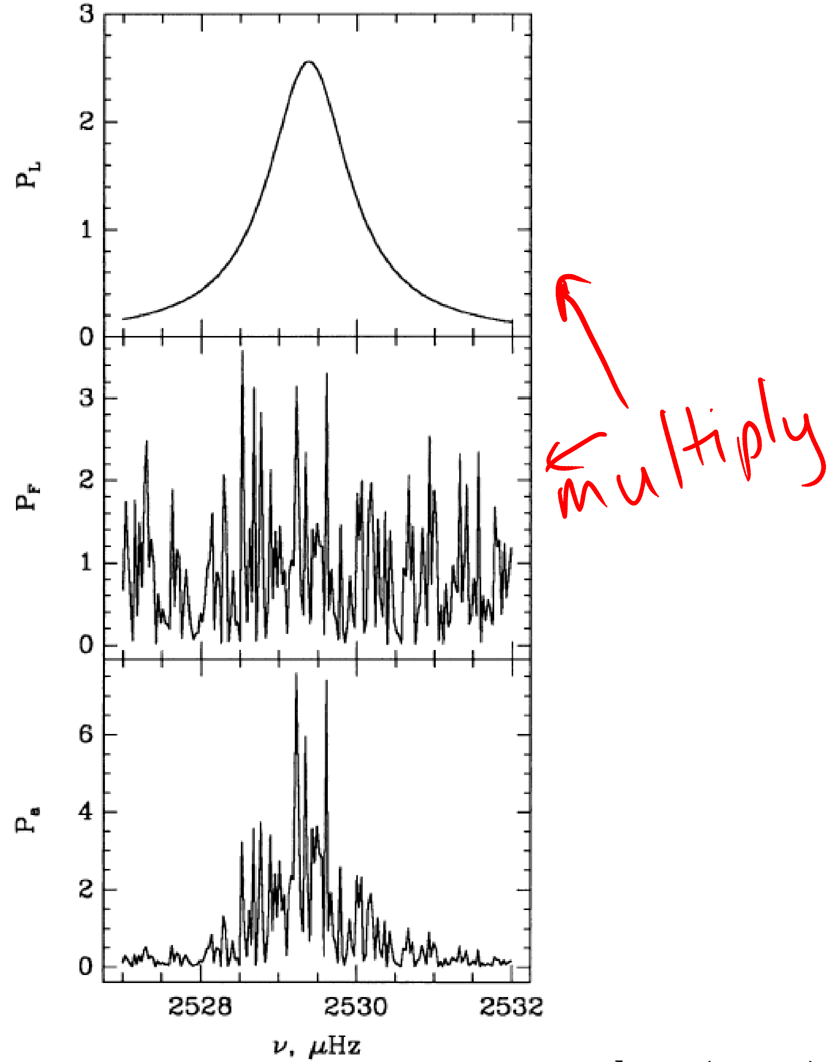
Green (VIRGO)
Amplitude (ppm)
7
6
5
4
3
2
1
0
3.0
3.2
3.4
3.6
3.8
4.0
Frequency (mHz)

20 days

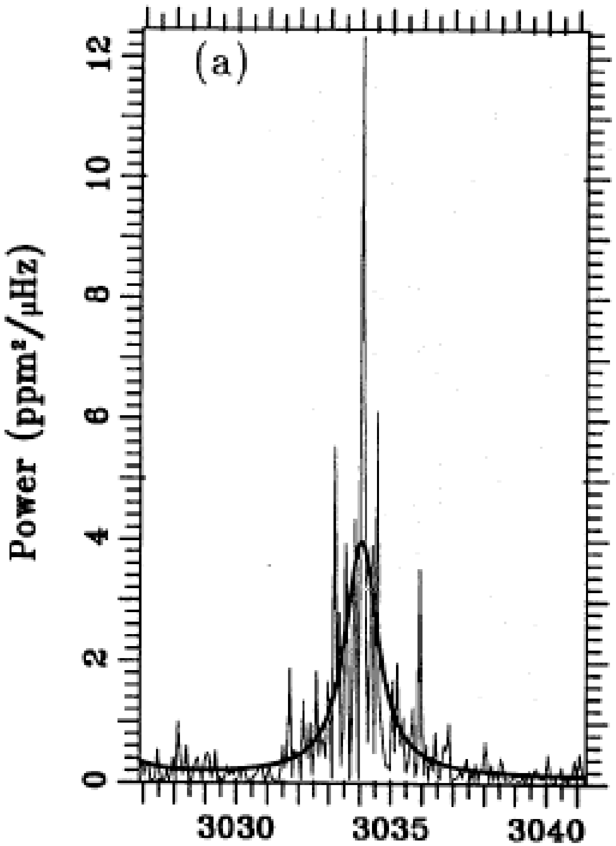


Modes are resolved into Lorentzian profiles

Solar-like oscillations are randomly excited and damped

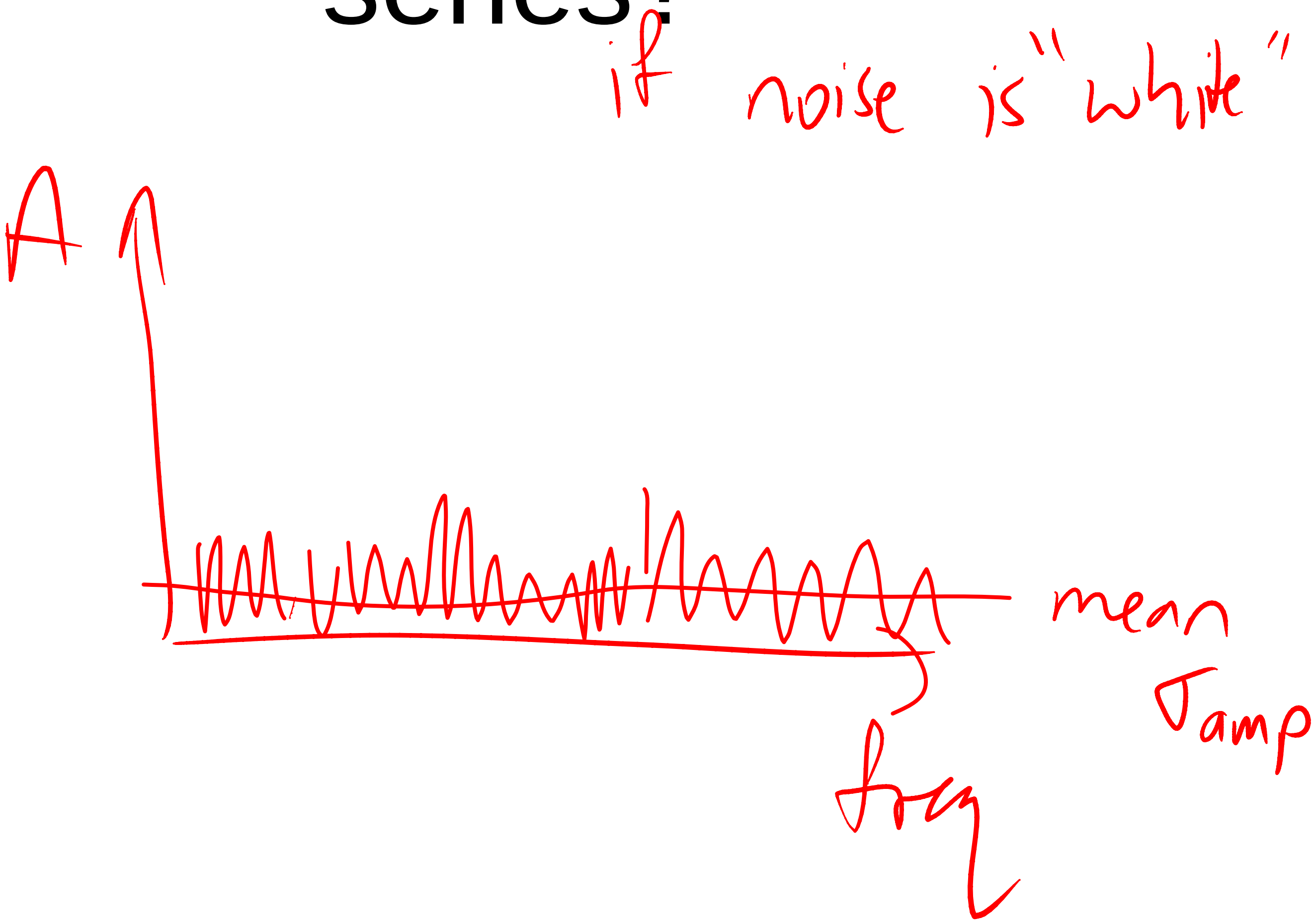


Kosovichev (1995)



Solar linewidths from IPHIR (Toutain & Frohlich 1992)

What happens when we add random noise to the time series?



from Kjeldsen & Bedding (1995):

1992). Firstly, the mean noise level in the power spectrum is

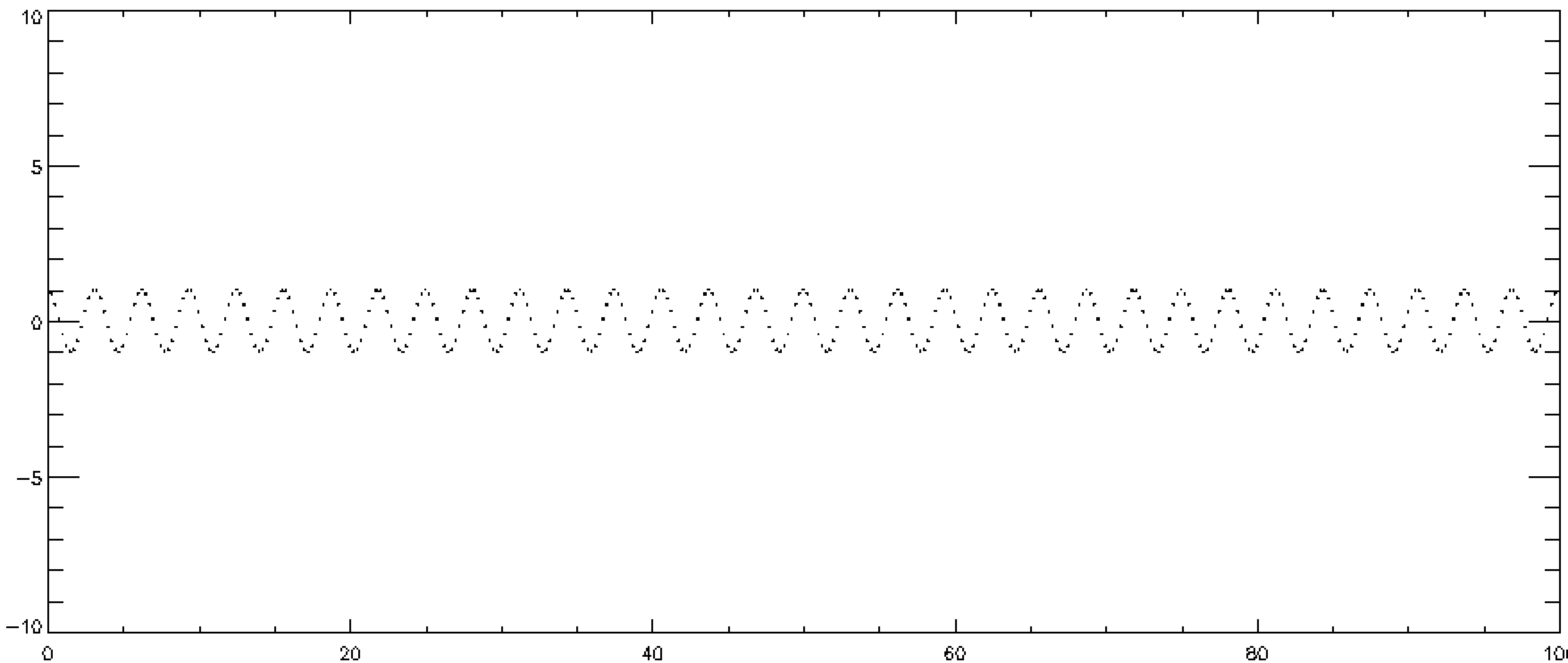
$$\sigma_{\rm PS} = 4\sigma_{\rm rms}^2/N, \quad \text{(A1)}$$

where N is the number of measurements in the time series and $\sigma_{\rm rms}$ is their rms scatter. Secondly, if the noise is gaussian then the mean noise level in the *amplitude* spectrum (which is the square root of the power spectrum) is:

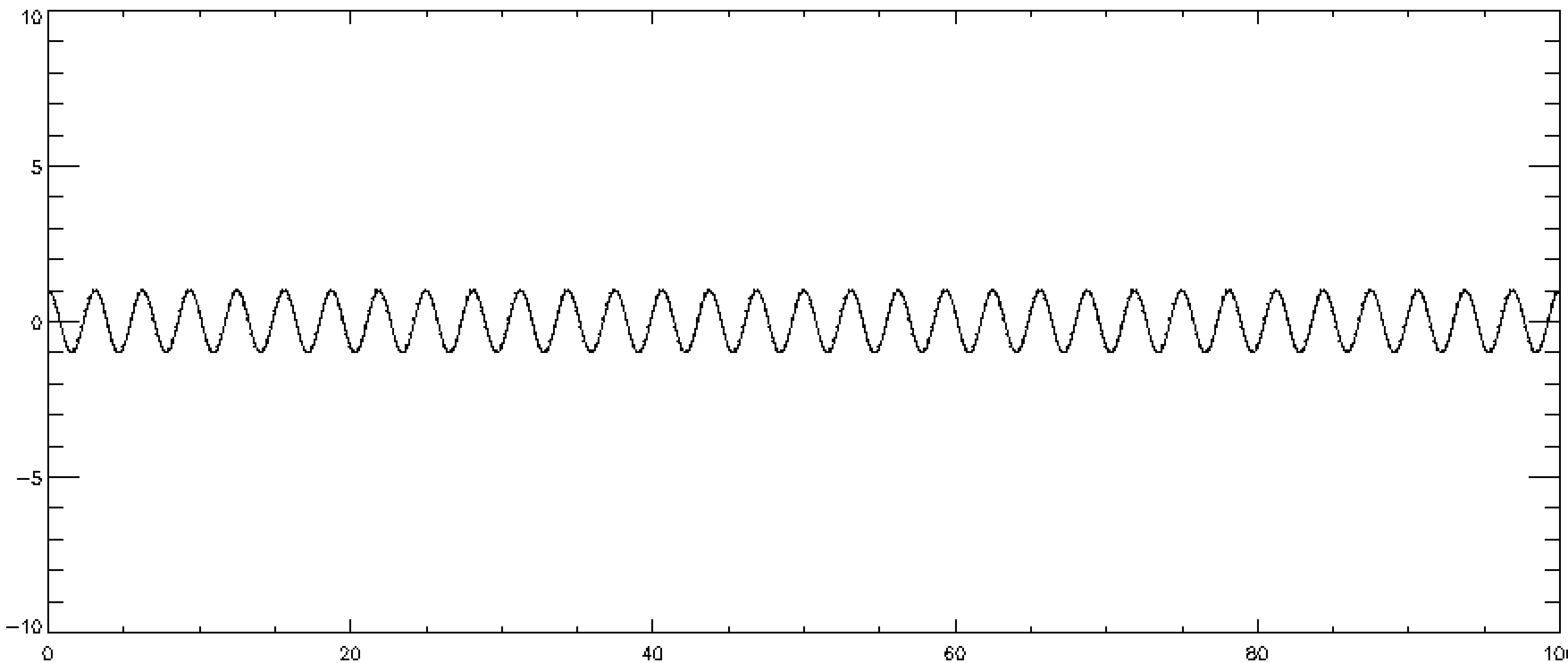
$$\sigma_{\rm amp} = \sqrt{\pi\sigma_{\rm PS}/4}. \quad \text{(A2)}$$

$$= \sigma_{\rm rms}\sqrt{\frac{\pi}{N}}$$

What happens when we have regular gaps in the time series?



10
5
0
−5
−10
0
20
40
60
80
100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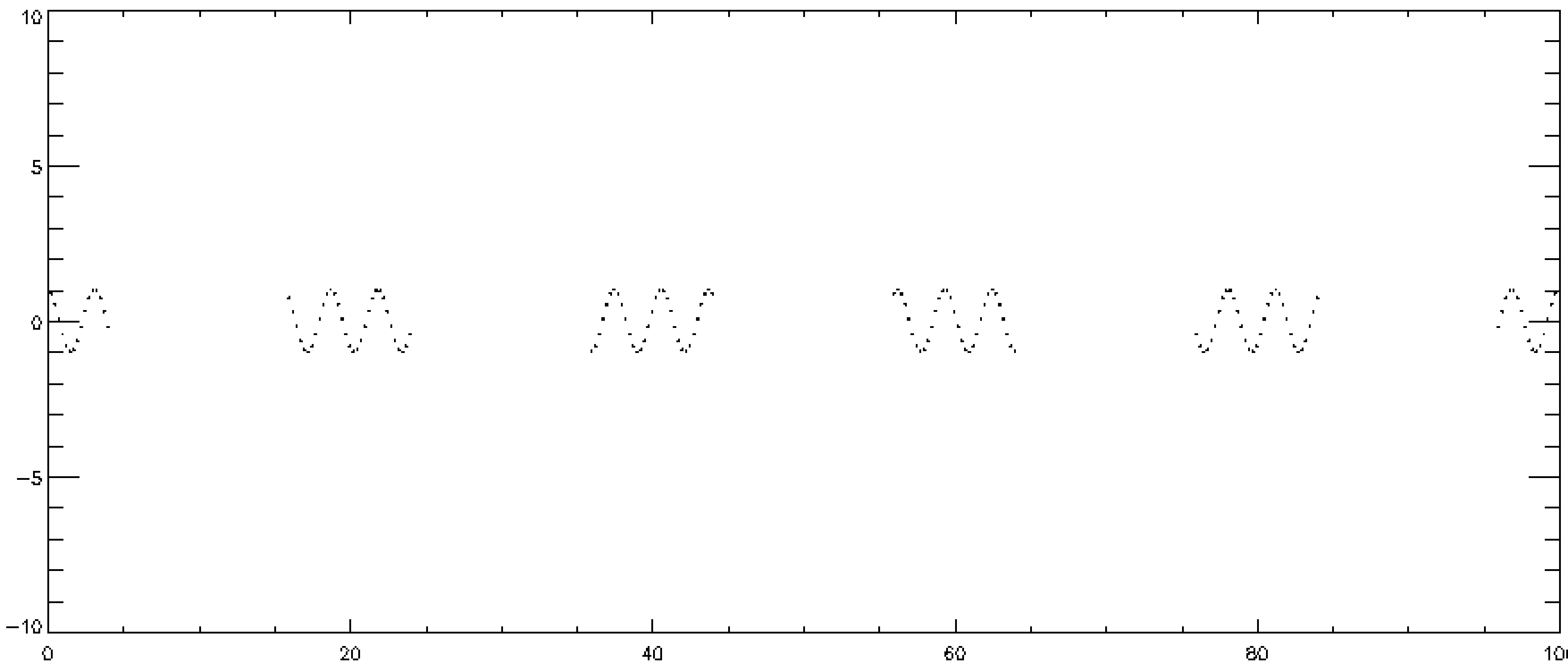
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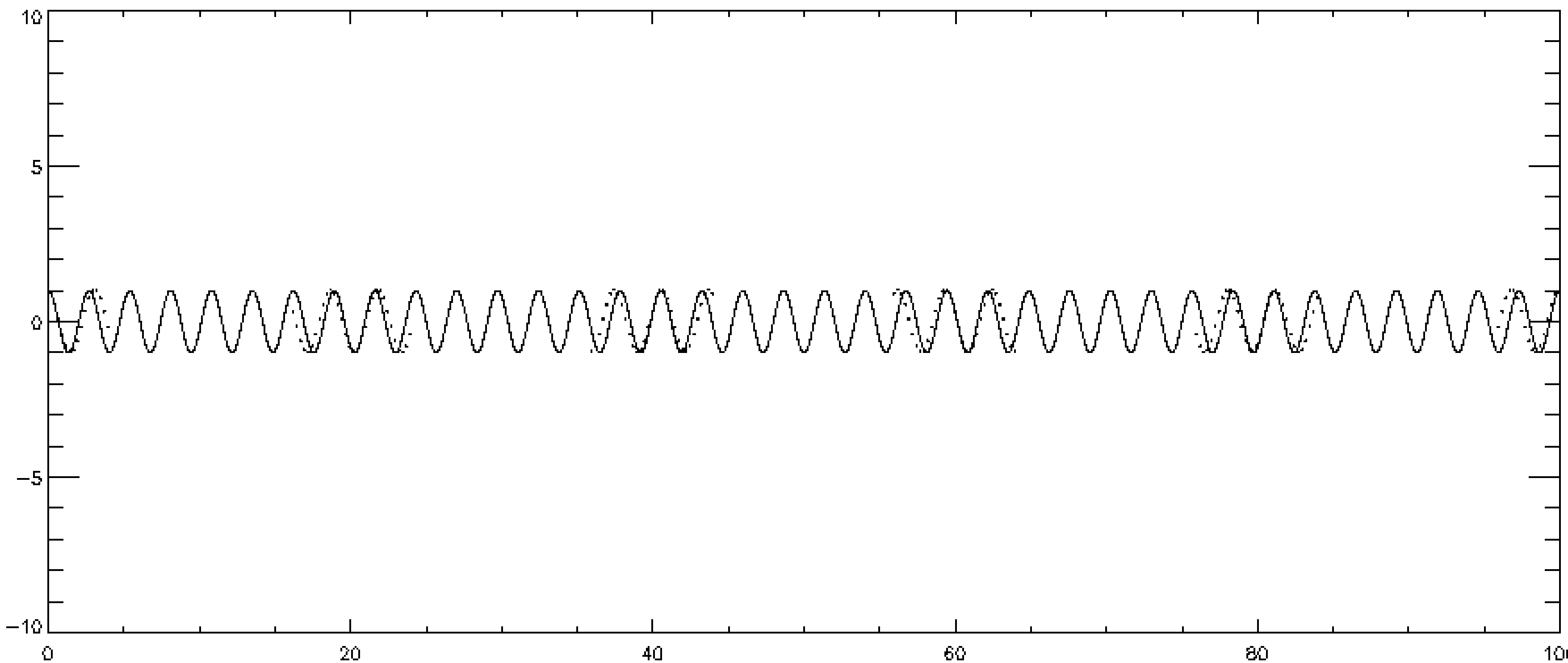

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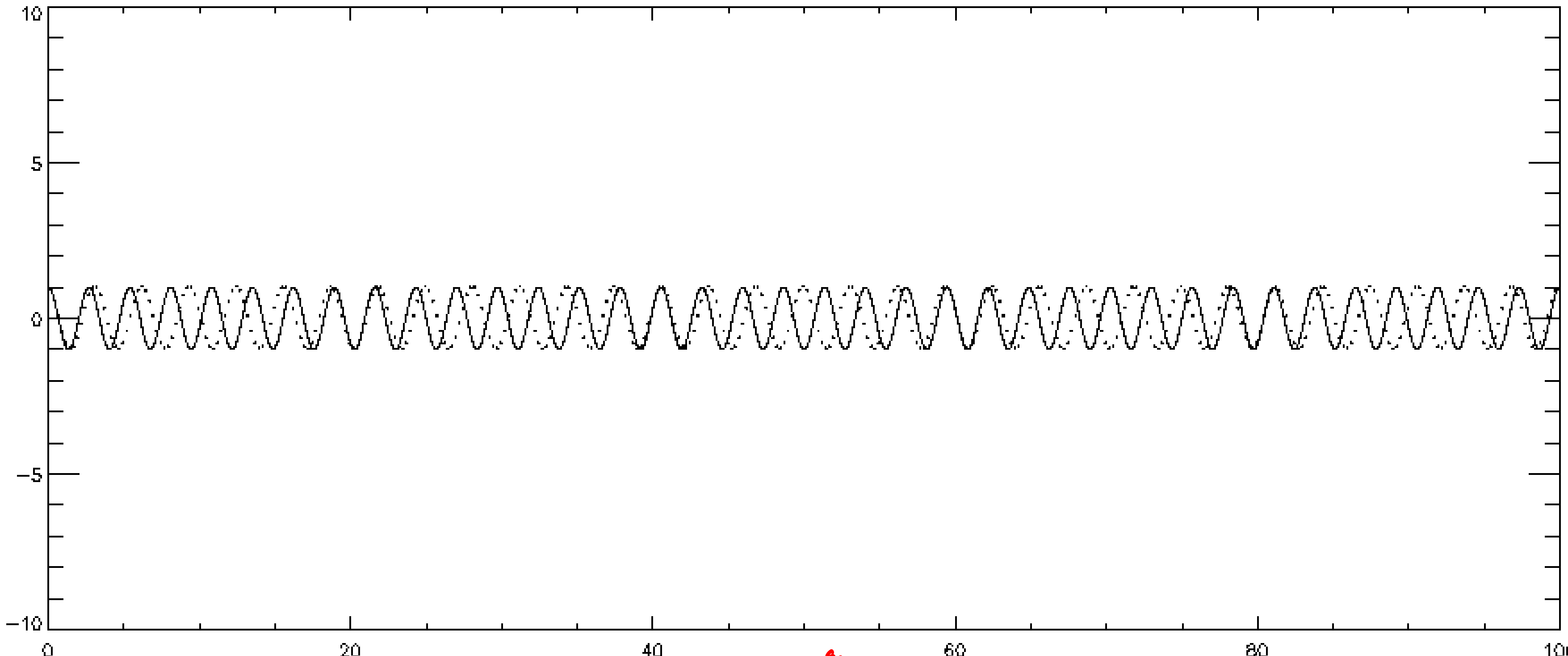
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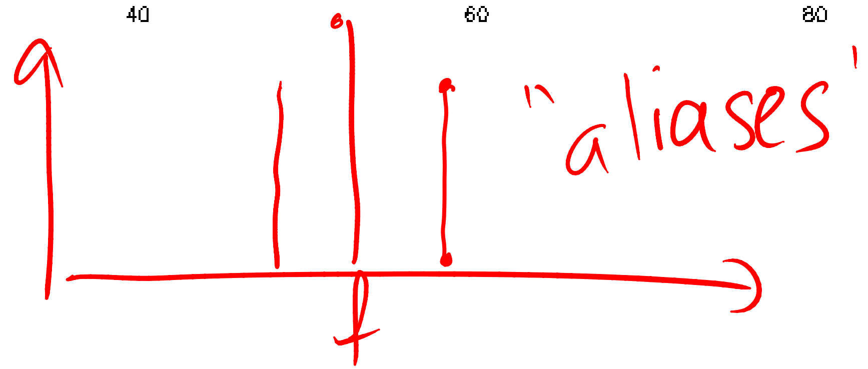
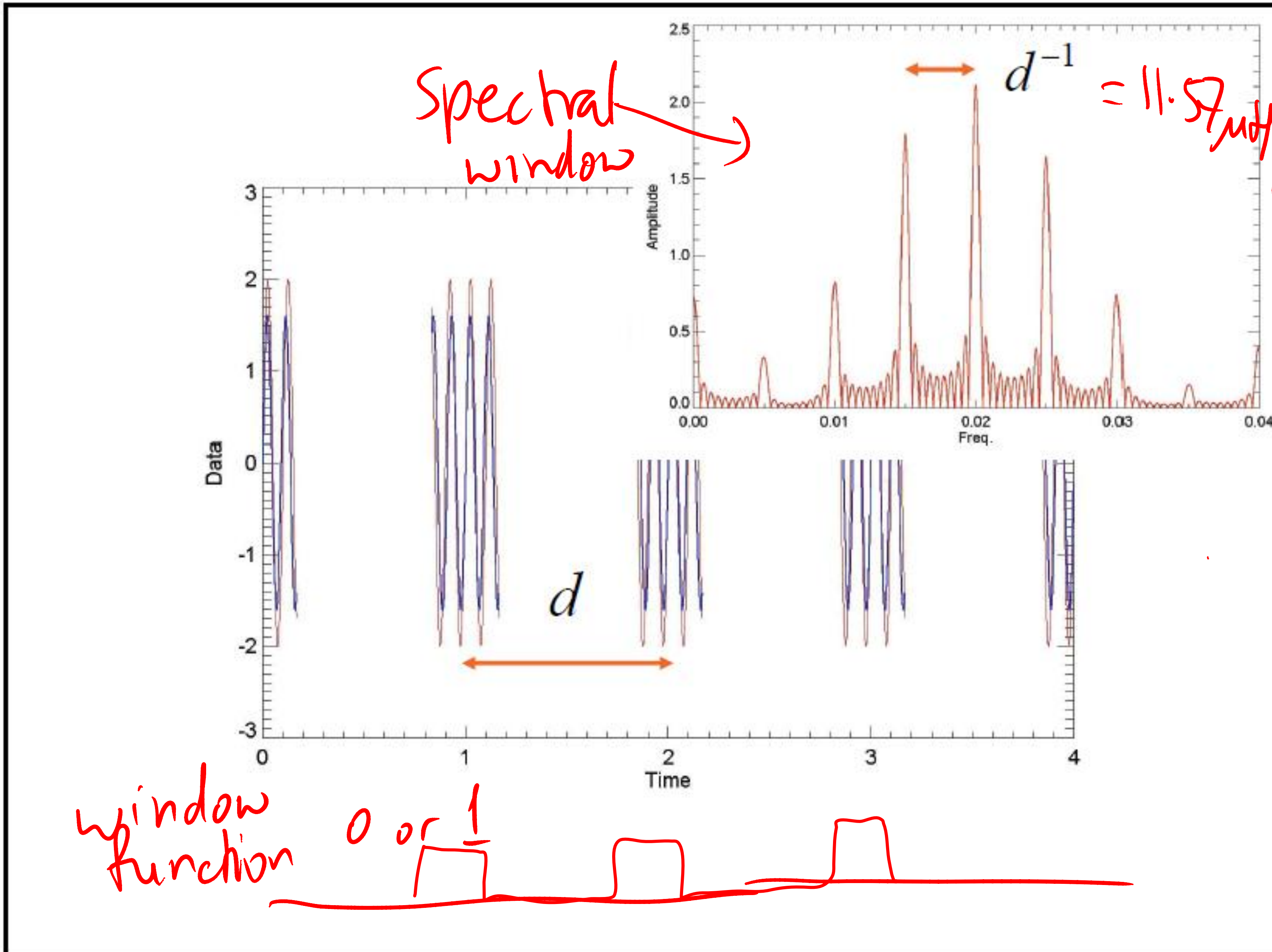
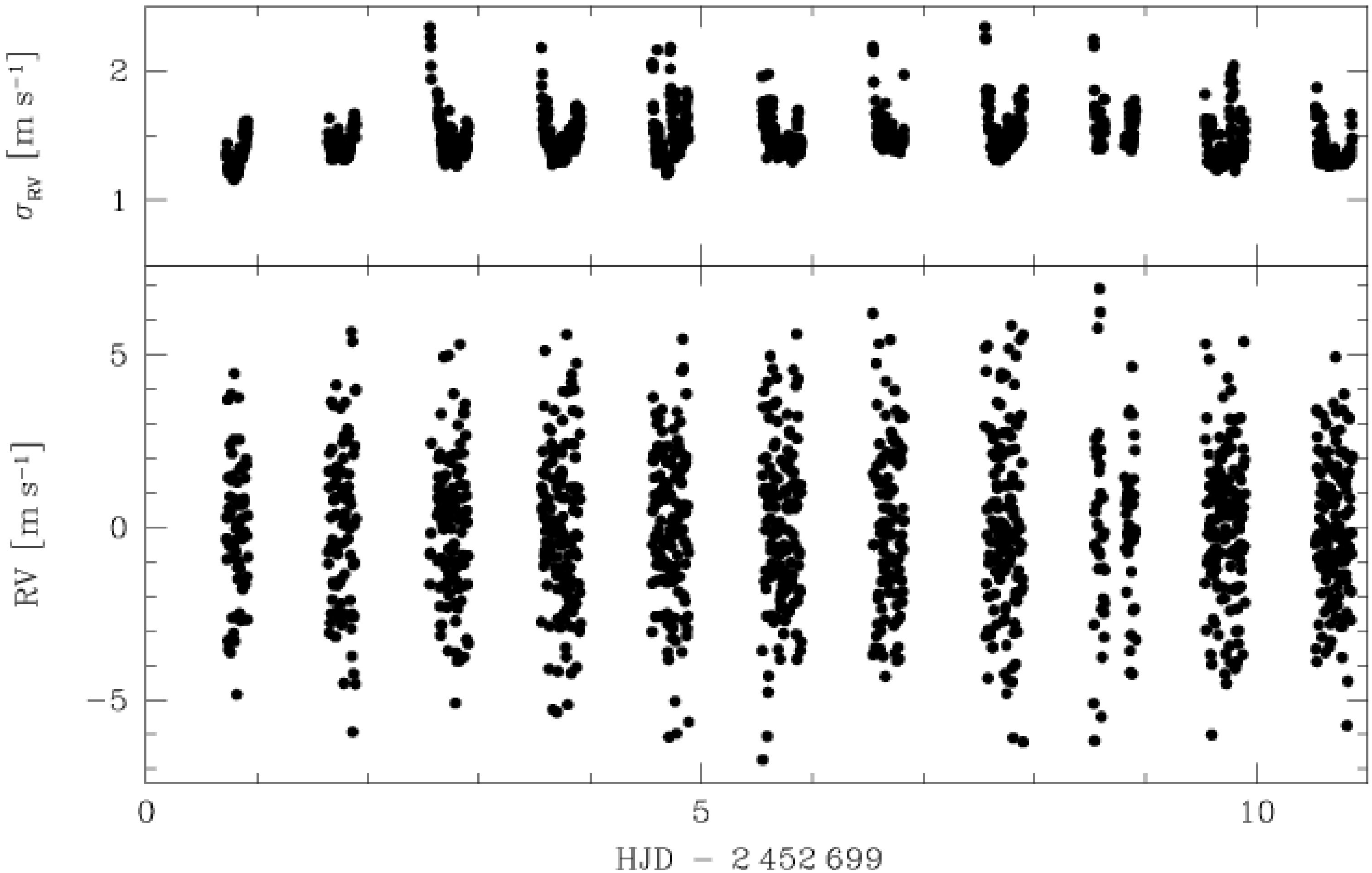

a
f
"aliases"



http://owww.phys.au.dk/~hans/tidsserie/Lecture_03.pdf

β Vir

Carrier et al. (2005): CORALIE



β Vir

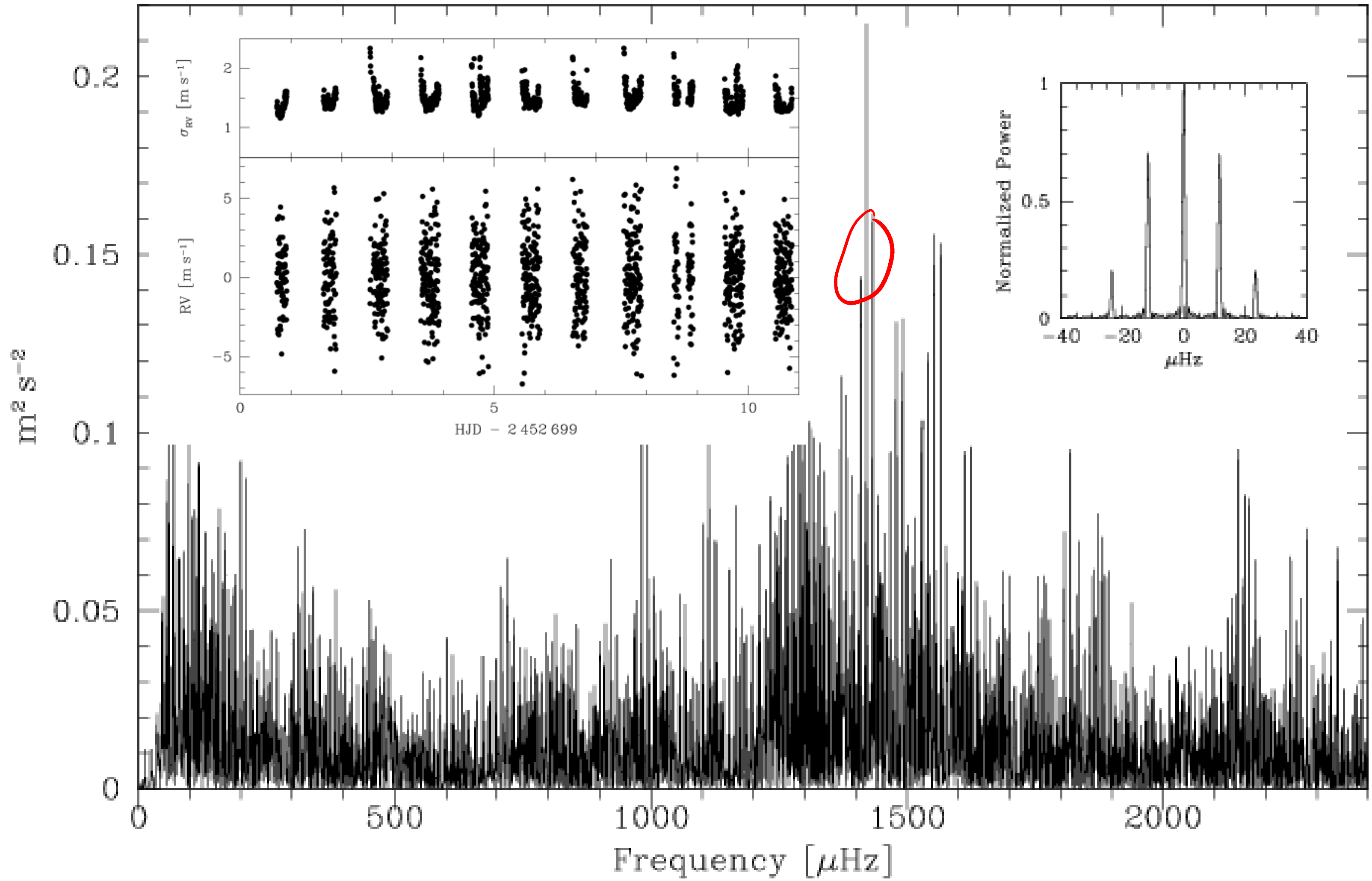
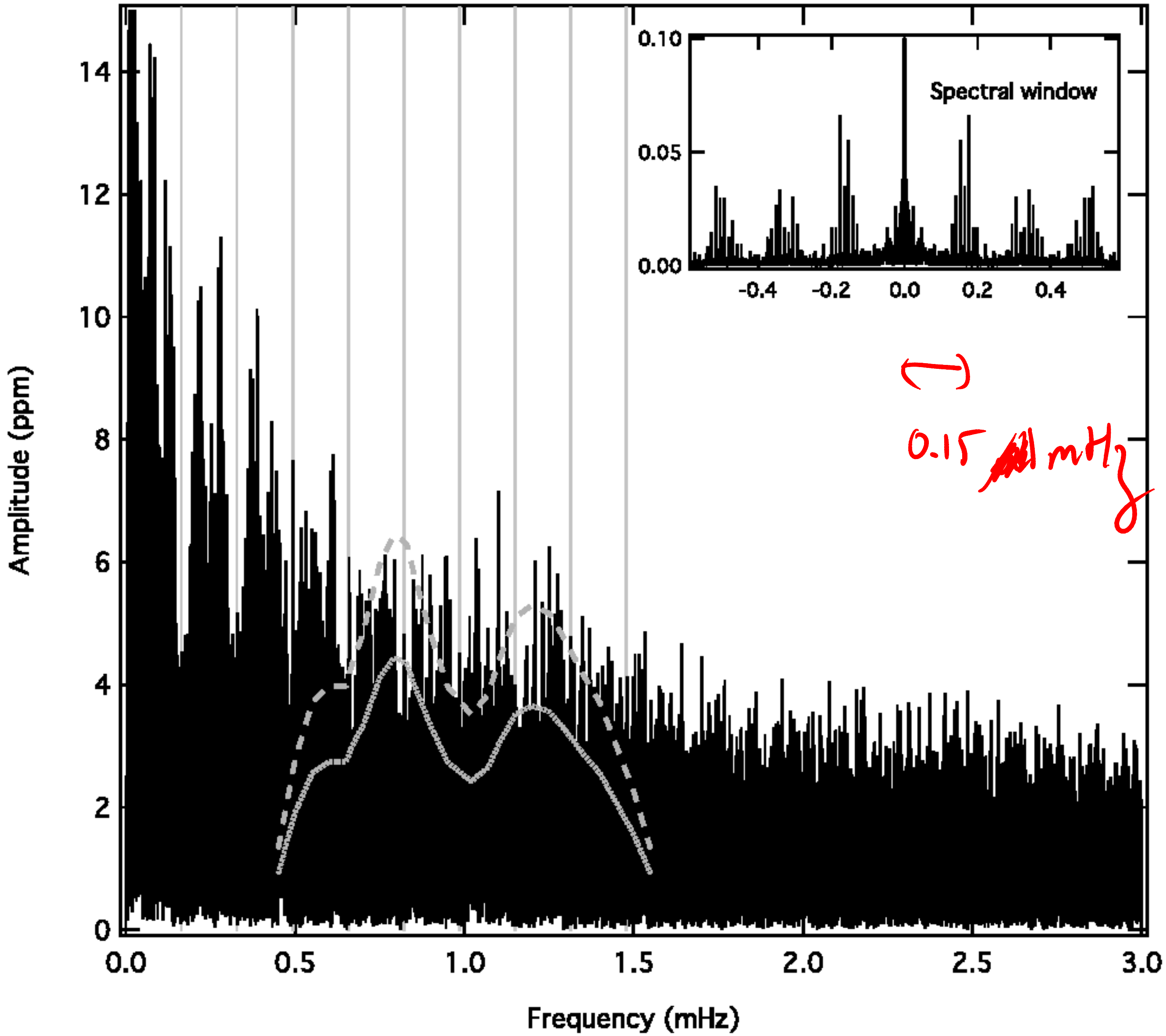


Fig. ~~2~~. Power spectrum of the CORALIE radial velocity measurements of β Vir. The ~~window function~~ is shown in the inset.

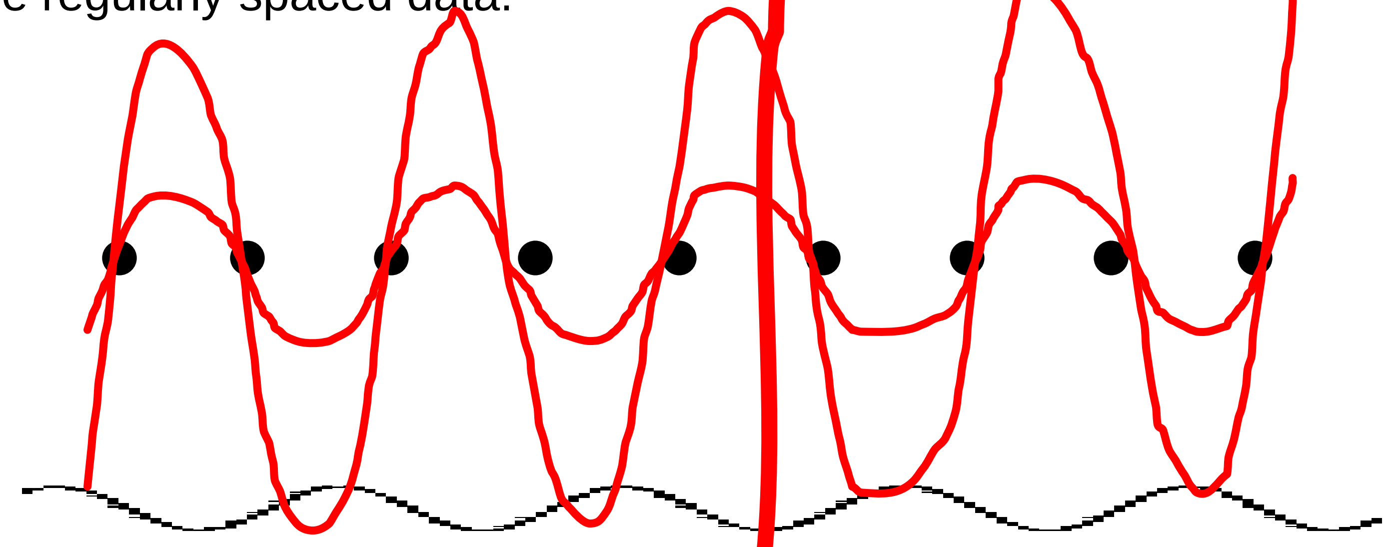
Spectral window



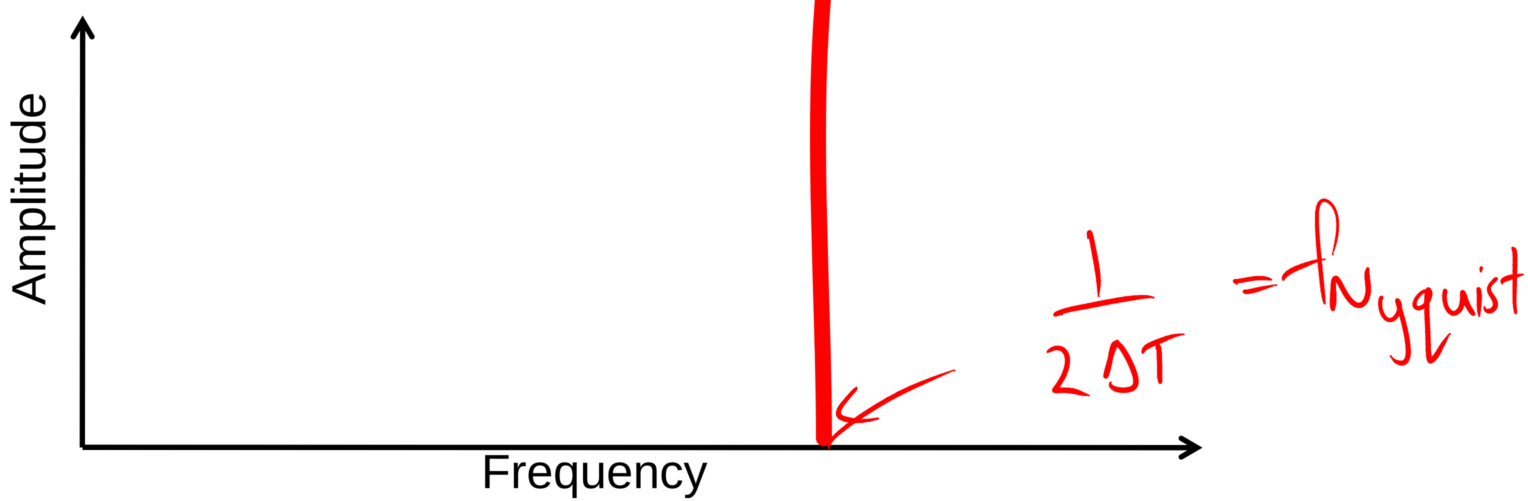
Procyon with *MOST*: Guenther et al. (2008)

What is this thing called "Nyquist frequency"?

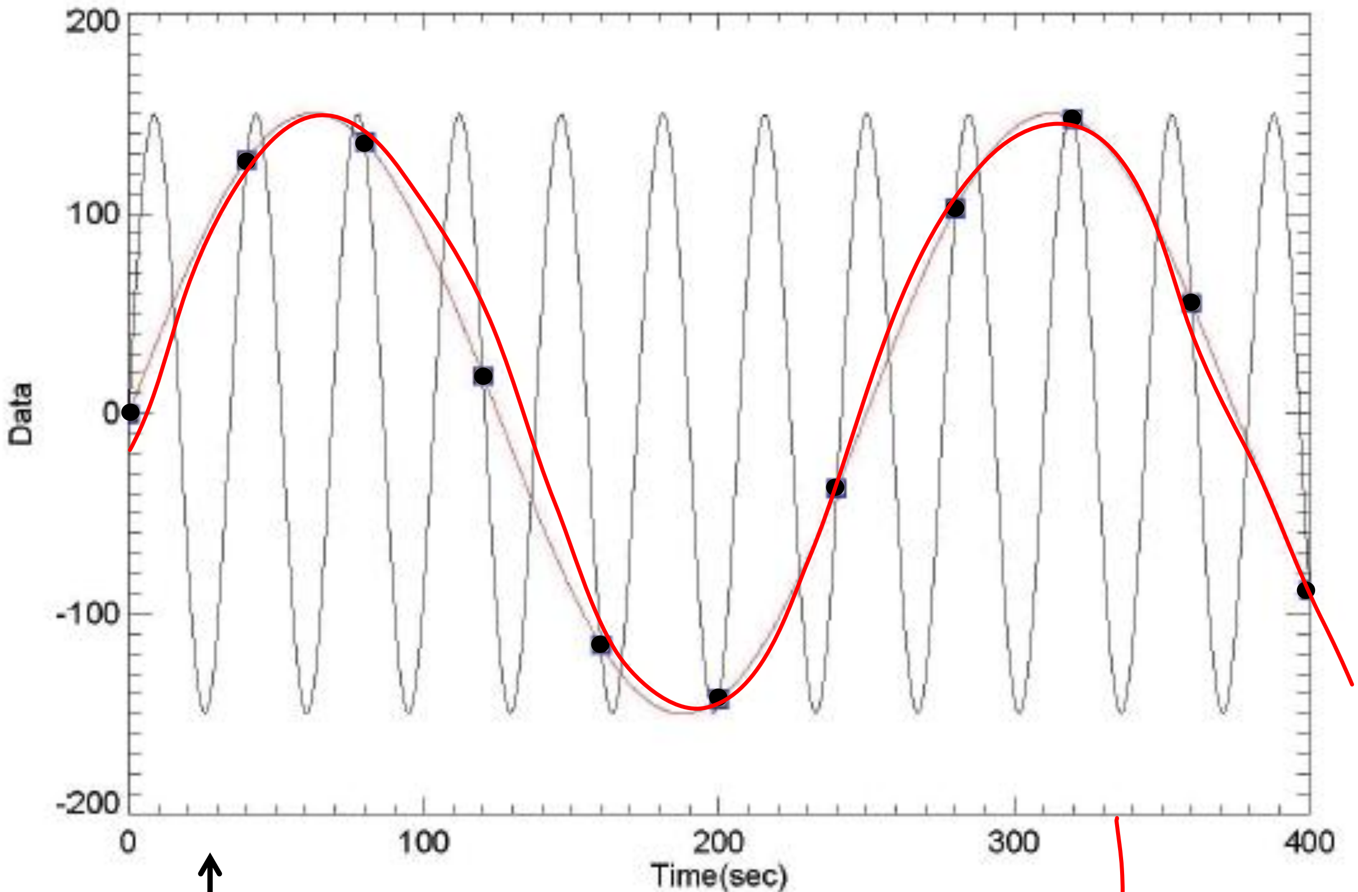
Suppose we have regularly spaced data:



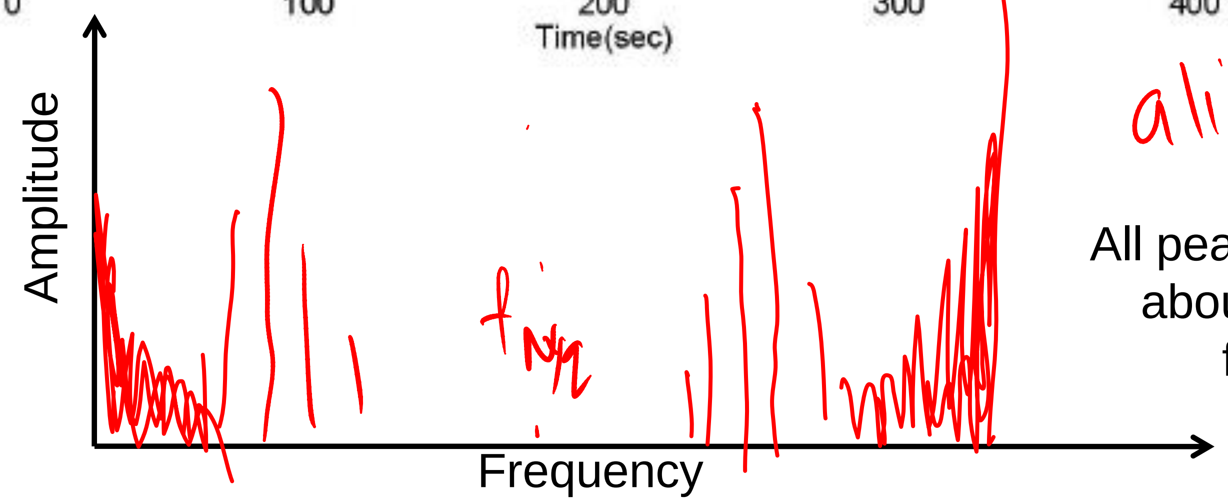
choose freq:



Some data:



a very high frequency also fits the data

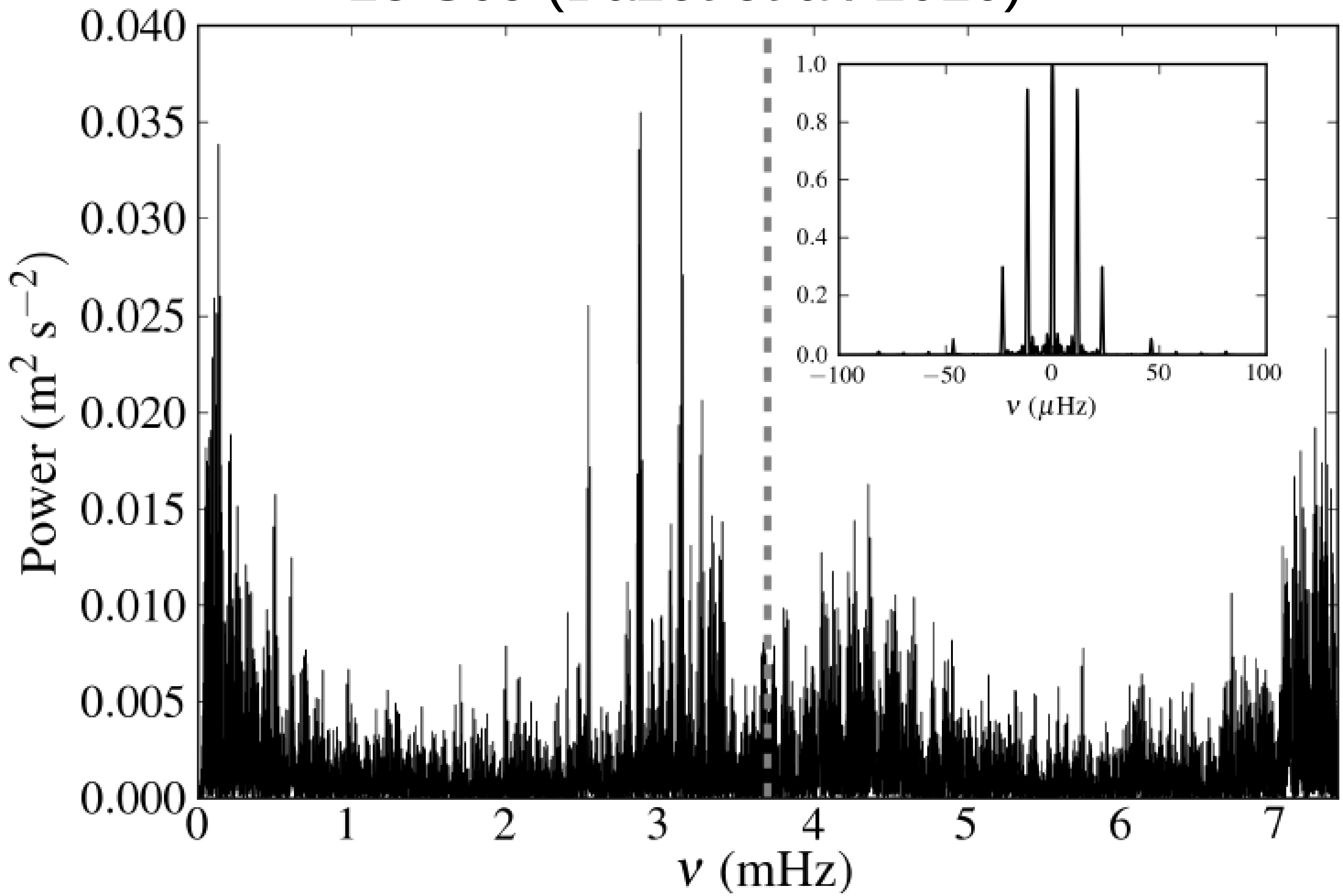


aliasing

All peaks are reflected about the Nyquist frequency

There is an effective Nyquist frequency, even if the observations are not exactly regularly spaced

18 Sco (Bazot et al. 2010)



Further reading

- "*The Fourier Transform & Its Applications*", book by Ronald N. Bracewell