Stellar structure and evolution

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Outline

1. The lives of stars
   – Overview of stellar evolution

2. Physics of stellar evolution
   – Stellar structure equations, time scales

3. Evolution of abundances
   – Stellar evolution: dredge-ups
   – Galactic chemical evolution


The lives of stars

• Low- and intermediate-mass stars
• Massive stars
• Cosmic cycle

based on Iben & Tutukov 1997, Sky & Telescope

"Many of the sky's most dramatic showpieces are but chapters in the lives of stars"
Star formation

- Protostar: completely convective gas core within gas-dust envelope
- Accretion and contraction for $10^5$ to $10^7$ years, until equilibrium state reached
  $\rightarrow$ main-sequence star
  - Gravity $\leftrightarrow$ gas pressure gradient
  - Surface energy lost $\leftrightarrow$ nuclear energy generated in core
- Literature on star formation: e.g.
  McKee & Ostriker 2007, ARA&A 45
Main-sequence phase – core hydrogen burning

• Facts

  ▪ Star formation produces more low-mass stars than massive stars
  ▪ Nuclear reaction rates depend on temperature, temperature determined by mass
    → MS lifetime depends on mass: $t_{\text{MS}} \propto M^{-3}$

• Consequences

  ▪ Only 5 percent of all stars formed up to now have evolved beyond main sequence
  ▪ The most massive stars have been formed and ”burnt out” throughout thousands of generations since Big Bang
Post-main sequence evolution for low- and intermediate-mass stars

- Mass between $0.7$ and $10 \, M_\odot$
- $80$ to $90$ percent of total lifetime on main sequence
- When hydrogen is used up in core, temperature is too low for helium burning
  - Core contraction
  - Hydrogen-shell burning
  - Envelope expands to more than $100 \, R_\odot$ → **red giant**
Post-main sequence evolution for low- and intermediate-mass stars

- Core temperature rises until helium nuclei begin to fuse into carbon and oxygen nuclei
- Duration of core helium burning phase between 10 and 25 percent of main-sequence lifetime
- When helium is used up in core
  → C-O core with hydrogen- and helium-burning shell
  → asymptotic giant branch (AGB) star
AGB stars are important for galactic chemical evolution

- About 97% of single stars ($M \gtrsim 0.7 \, M_\odot$) become AGB stars
- Produce half of carbon in universe
- Produce heavy elements by neutron capture
- Form dust grains in their atmospheres
- Strong winds carry processed gas and dust out into interstellar medium
- $\rightarrow$ provide the necessary conditions for the formation of future, more metal-rich generations of stars
Low- and intermediate-mass stars – white dwarfs

• At the end of the very short AGB phase, envelope matter is transferred into circumstellar space due to wind

• Remnant C-O core contracts to about size of Earth and temperature of 100,000 K → white dwarf

• UV and X-ray emission from white dwarf causes ejected gas to shine → planetary nebula

• White dwarf gradually cools to ≈4,000 K
Massive stars

- Mass between 10 and 50 $M_\odot$
- Short main-sequence phase $\rightarrow$ red or blue *supergiant* with He core
- He burning $\rightarrow$ C-O core
- Core contraction $\rightarrow$ carbon burning $\rightarrow$ oxygen-neon core $\rightarrow$ neon burning
- Several cycles of contraction – heating – ignition $\rightarrow$ *red supergiant with iron core*
A Red Supergiant’s Core

Diagram not to scale
Massive stars – core collapse

- When mass of iron core > 1.4 $M_\odot$
  - core collapses within fraction of second to size of small city
- Iron nuclei photodisintegrate $\rightarrow$ He nuclei $\rightarrow$ neutrons $\rightarrow$ **neutron star**
- Matter above core ejected with high velocities $\rightarrow$ **Type II supernova**
  - luminosity comparable to entire galaxy
- Chemical elements produced during explosion are returned to interstellar medium
Animation of supernova explosion dissolving into Chandra First Light Image, Cassiopeia A

http://chandra.harvard.edu/resources/animations/sn_to_casa_sm_web.mov
Galactic supernova remnants

Crab, 1000 years ago
Credit: FORS Team, 8.2-meter VLT, ESO

Vela, 10000 years ago

© Anglo-Australian Obs/Royal Obs. Edinburgh
Cosmic cycle

NGC 3603

HST • WFPC2

PRC99-20 • STScI OPO • June 1, 1999
Wolfgang Brandner (JPL/IPAC), Eva K. Grebel (Univ. Washington), You-Hua Chu (Univ. Illinois, Urbana-Champaign) and NASA
Physics of stellar evolution

- Equations of stellar structure and evolution
- Timescales of stellar evolution
- Evolutionary tracks
Inside the Sun?
Basic assumptions

• Star = radiating body of gas bound by self-gravity
• Radiation originates from internal energy sources
  ▪ Nuclear energy from fusion reactions
  ▪ Gravitational potential energy from contraction
• Star in empty space, not influenced by other stars
  ▪ Average distance between stars much larger than average star diameter
  ▪ Gravity and radiation decrease by factor $10^{-15}$ from one star to another
• Ignore deviations from spherical symmetry (e.g. rotation, magnetic field)
Basic assumptions

• Physical properties are uniform over spherical surface of radius $r$

• Local thermodynamic equilibrium (LTE)
  ▪ Average properties of the gas in volume element at $r$ are described by **local state variables**
  ▪ Temperature $T$, density $\rho$, pressure $P$, internal energy $u$, energy release rate $q$, luminosity $L$, chemical composition
  ▪ Changes of state variables are small over *mean free path* of particles and over *mean free time* between collisions between particles, including photons
Basic assumptions

- Initial **chemical composition** homogeneous as in Sun
  - **Number** abundances, from abundance analysis:
    \[
    [M_i/H] = \log\left[\frac{n(M_i)}{n(H)}\right] - \log\left[\frac{n(M_i)_{\odot}}{n(H)_{\odot}}\right]
    \]
  - **Mass** fractions: hydrogen $\rightarrow X$, helium $\rightarrow Y$, all metals (M) $\rightarrow$ **metallicity** Z, any species $M_i \rightarrow X_i$

\[
[M/H] \approx [Fe/H] \approx \log\left(\frac{Z}{Z_{\odot}}\right)
\]

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<th>[Fe/H]</th>
<th>Z</th>
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<td>0.014</td>
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<tr>
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Model of a star

- **Stellar structure** =
  State variables as a function of $r$ (distance from center)

- **Stellar evolution** =
  Change of structure with $t$ (time)

- Derive *equations for state variables* as functions of $r$ and $t$ from
  - conservation laws
  - nuclear physics (nuclear reaction rates)
  - description of energy transport (by radiation and convection)
  - atomic physics (opacity)
  - equation of state
Conservation of mass and momentum

- \( m(r) \) … mass contained in sphere with radius \( r \)
- \( dm \) … mass of shell between \( r \) and \( r+dr \)

\[
dm = 4\pi r^2 \rho dr
\]

- **Use** \( m \) **as independent variable**, because total stellar mass changes less during evolution than stellar radius
- **Conservation of momentum** \( \rightarrow \) equation for **pressure**
- Nuclear reaction rates \( \rightarrow \) equations for **mass fractions** of chemical species, \( X_i \)
Energy

• Conversation of energy and energy transport → equations for **luminosity** and **temperature**

• **Transport of energy** inside stars by
  1. Conduction (collisions between particles)
     - cores of evolved stars, white dwarfs, neutron stars
  2. **Convection**
     - occurs in case of intense nuclear burning or high opacity
     - important effects: energy transport and **mixing**
  3. **Radiation**
     - diffusion approximation
Radiative transport

- **Mean free path** of photons:
  \[ \ell_{\text{ph}} = \frac{1}{\kappa \rho} \]
  \( \kappa \) ... radiative cross-section per unit mass (absorption coefficient, **opacity**)
  typical values: \( \kappa = 1 \text{ cm}^2 \text{ g}^{-1}, \rho = 1 \text{ g cm}^{-3} \rightarrow \ell_{\text{ph}} = 1 \text{ cm} \)

- Compared to stellar radius, \( \ell_{\text{ph}} \) is small \( \rightarrow \) can treat radiative transport as a **diffusion process**
  = random walk of photons

- How long is the travel time \( t_{\text{ph}} \) of photons from the interior to the surface of the Sun? Notes: distance travelled \( \approx \) step size times square root of number of steps;
Radiative transport

• How long is the travel time $t_{\text{ph}}$ of photons from the interior to the surface of the Sun?

Number of steps after which distance $R_{\odot}$ is reached:

$$N = \left( \frac{R_{\odot}}{\ell_{\text{ph}}} \right)^2$$

$$\Rightarrow t_{\text{ph}} = \frac{N \ell_{\text{ph}}}{c} \approx \frac{(7 \cdot 10^{10})^2}{3 \cdot 10^{10} \cdot 3 \cdot 10^7} \approx 5000 \text{ yr}$$

$l_{\text{ph}} = 1 \text{ cm}$

Mitalas & Sills 1992: $\langle l_{\text{ph}} \rangle = 0.09 \text{ cm}$, $t_{\text{ph}} = 170000 \text{ yr}$
Opacity from atomic physics

• Absorption coefficient calculated for all possible interactions between particles and photons
• Tabulated for different combinations of $T$, $\rho$ and $X_i$
• Or approximated by power law:
  \[ \kappa = \kappa_0 \rho^a T^b \]
• Most important interactions:
  electron scattering – $a=0$, $b=0$; free-free absorption – $a=1$, $b=-7/2$
• Opacity tables
  ▪ OPAL (Rogers and Iglesias), also equation of state tables: http://rdc.llnl.gov/opal.html
  ▪ Opacity Project (International collaboration) http://cdsweb.u-strasbg.fr/topbase/TheOP.html

Rosseland opacity

\[ R = \frac{\rho}{(10^{-6}T)^3} \]
Equations of stellar structure and evolution

\[ \frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \]

\[ \frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2} \ddot{r} \]

\[ \frac{\partial L}{\partial m} = q - \dot{u} - P \left( \frac{1}{\rho} \right) \]

\[ \frac{\partial T}{\partial m} = -\frac{3}{16\sigma} \frac{\kappa}{T^3} \frac{L}{(4\pi r^2)^2} \]

\[ \frac{\partial X_i}{\partial t} = f_i(\rho, T, \vec{X}) \]

\[ \kappa = \kappa_0 \rho^a T^b, \quad q = q_0 \rho^m T^n, \quad P = f(\rho, T, X_i) \]
Equations of stellar structure and evolution

\[
\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \\
\frac{\partial P}{\partial m} = - \frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2 \ddot{r}} \\
\frac{\partial L}{\partial m} = q - \dot{v} - P \left( \frac{1}{\rho} \right) \\
\frac{\partial T}{\partial m} = - \frac{3}{16\sigma} \frac{\kappa}{T^3} \frac{L}{(4\pi r^2)^2} \\
\frac{\partial X_i}{\partial t} = f_i (\rho, T, \bar{X})
\]

Equilibrium
\rightarrow \text{stellar structure equations}

Equation of state
\[ \kappa = \kappa_0 \rho^a T^b, \quad q = q_0 \rho^m T^n, \quad P = f (\rho, T, X_i) \]
Characteristic timescales of stellar evolution

- Equations of stellar evolution describe changes for three different properties on different timescales
- Structural changes $\rightarrow$ dynamical timescale

$$t_{\text{dyn}} = \frac{R}{\dot{R}} \approx \frac{R}{v_{\text{ff}}} = \left( \frac{R^3}{2GM} \right)^{1/2} \approx \left( \frac{1}{G\bar{\rho}} \right)^{1/2}$$

- **Sun**: $t_{\text{dyn}} \approx 15$ minutes, **red giants** ($100R_\odot$): 2 weeks, **white dwarfs** ($R_\odot/50$): 3 seconds
- $t_{\text{dyn}}$ = typical timescale on which star reacts to perturbation of hydrostatic equilibrium
Characteristics timescales of stellar evolution

- Change of internal energy $U_{\text{tot}} \rightarrow \text{thermal timescale}$
  - Virial theorem: $|U_{\text{tot}}| = \frac{1}{2} \Omega$
    
    Change in $U_{\text{tot}} = $ luminosity

    $$t_{\text{th}} = \frac{U_{\text{tot}}}{L} \approx \frac{GM^2}{2RL}$$

- **Sun**: $t_{\text{th}} \approx 15$ million years

- Much longer than $t_{\text{dyn}}$, but much shorter than stellar lifetime
Leads to stars mostly in thermal equilibrium

- $t_{\text{th}} =$ time a star could maintain constant luminosity by contraction = *Kelvin-Helmholtz timescale*
Characteristic timescales of stellar evolution

- Change in composition and "nuclear potential energy" $E_n$  
  $\rightarrow$ **nuclear timescale**
  - $E_n$ given by mass excess of hydrogen per unit mass times hydrogen mass fraction times stellar mass
  - Change in $E_n = $ nuclear luminosity $L_{\text{nuc}} = L$ in thermal equ.
    
    $$ t_{\text{nuc}} \approx \frac{7\text{MeV}/m_HXM}{L} $$

- **Sun**: $t_{\text{nuc}} \approx 100$ billion years, much larger than its age
- Stars don’t consume all available nuclear energy
- Only fraction of stellar mass changes composition
Characteristic timescales of stellar evolution

\[ t_{\text{dyn}} \ll t_{\text{th}} \ll t_{\text{nuc}} \]

→ rates of nuclear processes determine stellar evolution
→ can decouple equation for chemical composition from the others

1. Solve stellar structure equations for given composition
2. Apply time step and determine new composition
Solving for stellar structure

- **Four** differential equations with **boundary conditions** at
  - center of star \((m = 0)\): \(r = 0, L = 0\)
  - surface of star \((m = M)\): fit interior solution to a stellar atmosphere model

- Three "material functions" (for \(\rho, \kappa, q\))

- **Input** parameters:
  - mass \(M\), chemical composition \(X(t), Y(t), Z(t)\)

- **Output**: \(r(m), P(m), L(m), T(m), \rho(m), \kappa(m), q(m)\), for each time \(t\), in particular \(T_{\text{eff}}, L, R, \rho_c, P_c\)

- Equations are highly non-linear and coupled \(\rightarrow\) have to be solved with numerical methods
Examples of stellar evolution models and codes

• Geneva grids of stellar evolution models
database of models for masses between 0.8 and 120 solar masses, pre-main-sequence, and from main sequence up to carbon burning

• Padova database of stellar evolutionary tracks
http://stev.oapd.inaf.it/

• Dartmouth Stellar Evolution Web Server
Java applet for calculation of models between 0.5 and 5 solar masses, from main sequence to core helium burning
http://stellar.dartmouth.edu

• CESAM (Nice)
Code d'Évolution Stellaire Adaptatif et Modulaire
http://www-n.oca.eu/cesam/
Evolutionary tracks in theoretical HR diagram

- **Main sequence**
  1→2: Core H burning
  2→3: $X_{\text{center}}$ 0.05→0

- **Shell H burning (3→5)**
  - intermediate masses ($\geq 2 M_\odot$): fast → “Hertzsprung gap”
  - low masses: slow

- **Red giant branch (RGB, 5→6)**

- **Helium ignition in core at 6**

$$\log \left( \frac{L}{L_\odot} \right)$$

$\log T_{\text{eff}}$

- **9 $M_\odot$**
- **3 $M_\odot$**
- **1 $M_\odot$**
Summary of Part 2

- Assuming stars to be radiating gas spheres and applying basic physics of gas and radiation leads to differential equations for stellar structure variables.
- Estimation of characteristic timescales allows to decouple equations and solve for changes in space and time separately.
- Stellar evolution models predict surface properties of stars of different mass and age, which can be compared to observations.