Stellar structure and evolution

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Outline

1. The lives of stars
   – Overview of stellar evolution
2. Physics of stellar evolution
   – Stellar structure equations, time scales
3. Evolution of abundances
   – Stellar evolution: dredge-ups
   – Galactic chemical evolution

Selected literature

- Dina Prialnik, *An Introduction to the Theory of Stellar Structure and Evolution*, 2nd Edition, Cambridge University Press, 2009.
- Maurizio Salaris and Santi Cassisi, *Evolution of Stars and Stellar Populations*, John Wiley & Sons, 2005.
- B.E.J. Pagel, *Nucleosynthesis and Chemical Evolution of Galaxies*, 2nd Edition, Cambridge University Press, 2009.
- …

The lives of stars

- Low- and intermediate-mass stars
- Massive stars
- Cosmic cycle

based on Iben & Tutukov 1997, Sky & Telescope

”Many of the sky's most dramatic showpieces
are but chapters in the lives of stars”

Star formation

- Protostar: completely convective gas core within gas-dust envelope
- Accretion and contraction for 10^5 to 10^7 years, until *equilibrium state* reached → ***main-sequence star***
  - Gravity ↔ gas pressure gradient
  - Surface energy lost ↔ nuclear energy generated in core
- Literature on star formation: e.g.
  Science 2002, vol. 295, no. 5552
  McKee & Ostriker 2007, ARA&A 45


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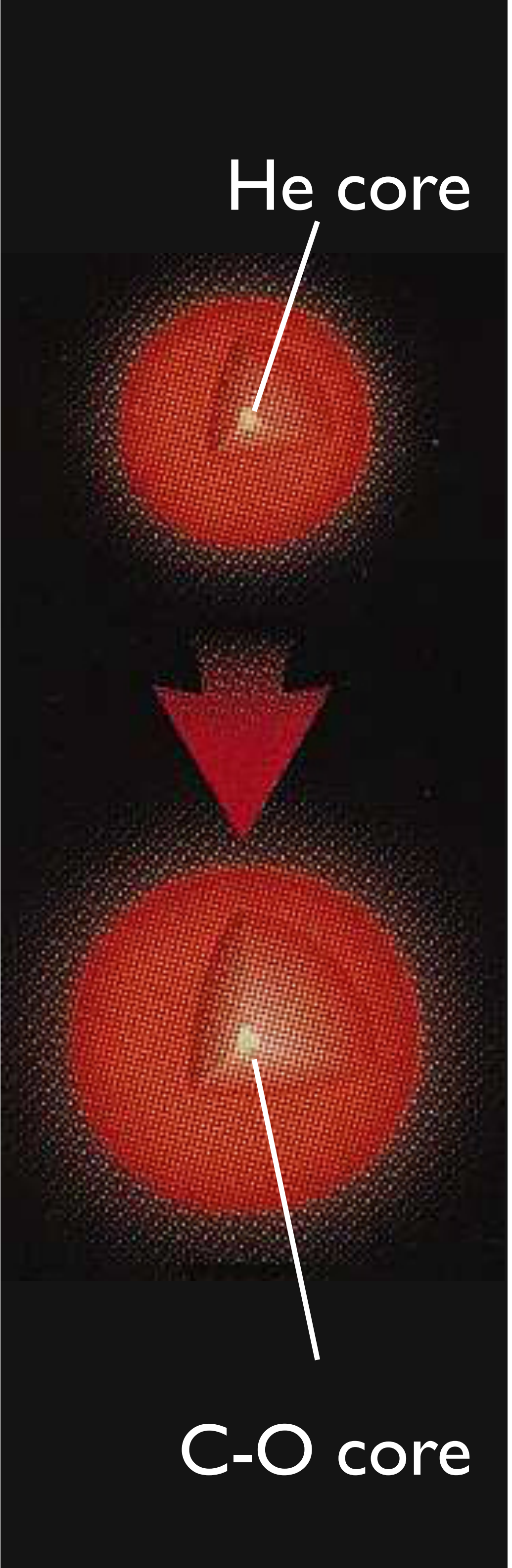
Main-sequence phase − core hydrogen burning

- Facts
  - Star formation produces more low-mass stars than massive stars
  - Nuclear reaction rates depend on temperature, temperature determined by mass
    → MS lifetime depends on mass: $\mathbf{t_{MS} \propto M^{-3}}$
- Consequences
  - Only 5 percent of all stars formed up to now have evolved beyond main sequence
  - The most massive stars have been formed and ”burnt out” throughout thousands of generations since Big Bang

Post-main sequence evolution for low- and intermediate-mass stars

- Mass between 0.7 and 10 $M_\odot$
- 80 to 90 percent of total lifetime on main sequence
- When hydrogen is used up in core, temperature is too low for helium burning
  - Core contraction
  - Hydrogen-shell burning
  - Envelope expands to more than 100 $R_\odot$ $\rightarrow$ ***red giant***



Post-main sequence evolution for low- and intermediate-mass stars

- Core temperature rises until helium nuclei begin to fuse into carbon and oxygen nuclei
- Duration of **core helium burning** phase between 10 and 25 percent of main-sequence lifetime
- When helium is used up in core
  → C-O core with hydrogen- and helium-burning shell
  → ***asymptotic giant branch (AGB) star***

AGB stars are important for galactic chemical evolution

- About 97% of single stars ($M \gtrsim 0.7\ M_\odot$) become AGB stars
- Produce half of carbon in universe
- Produce heavy elements by neutron capture
- Form dust grains in their atmospheres
- Strong winds carry processed gas and dust out into interstellar medium
- $\rightarrow$ provide the necessary conditions for the formation of future, more metal-rich generations of stars

Low- and intermediate-mass stars – white dwarfs

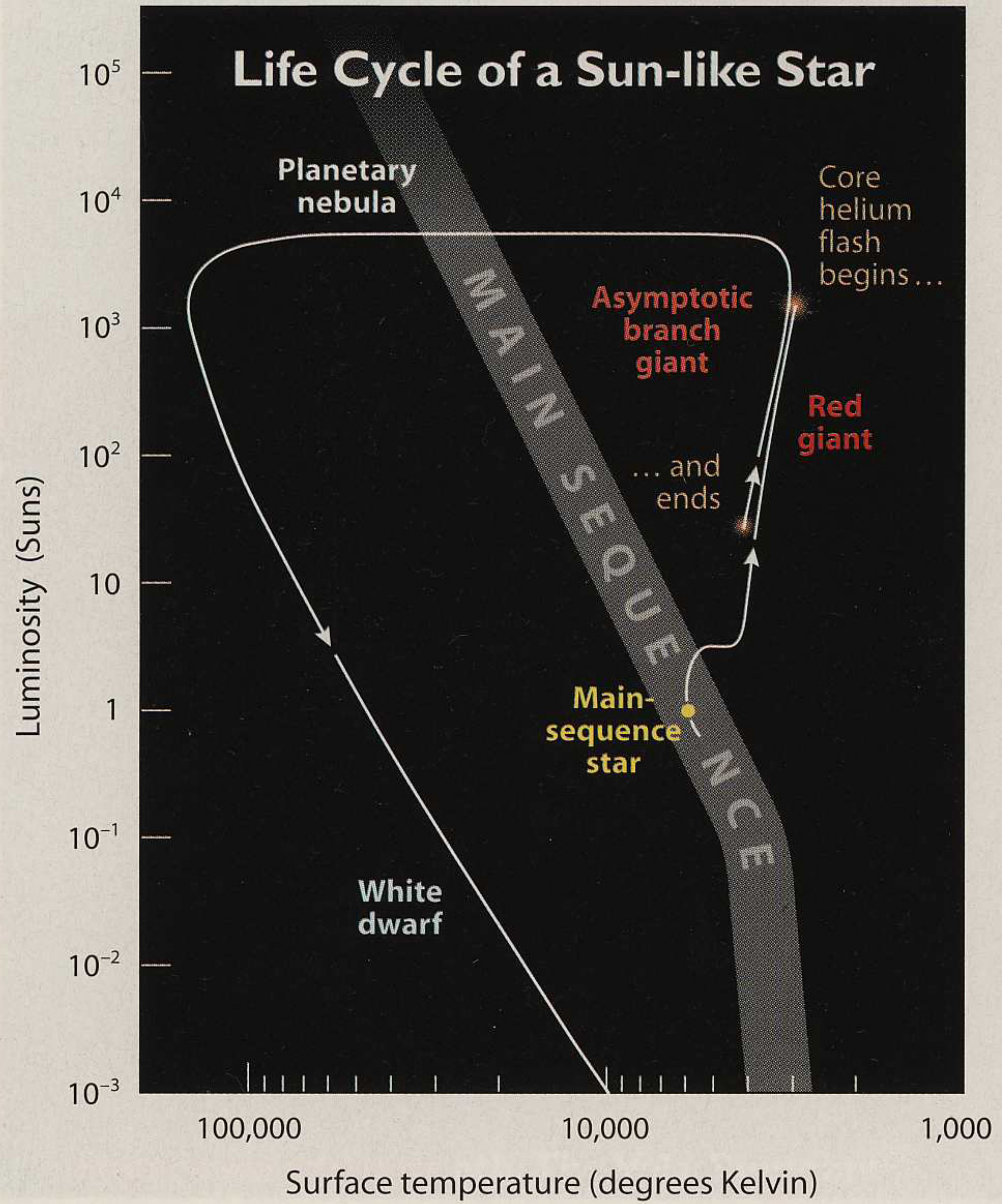
- At the end of the very short AGB phase, envelope matter is transferred into circumstellar space due to wind
- Remnant C-O core contracts to about size of Earth and temperature of 100,000 K
  → ***white dwarf***
- UV and X-ray emission from white dwarf causes ejected gas to shine
  → ***planetary nebula***
- White dwarf gradually cools to ≈4,000 K

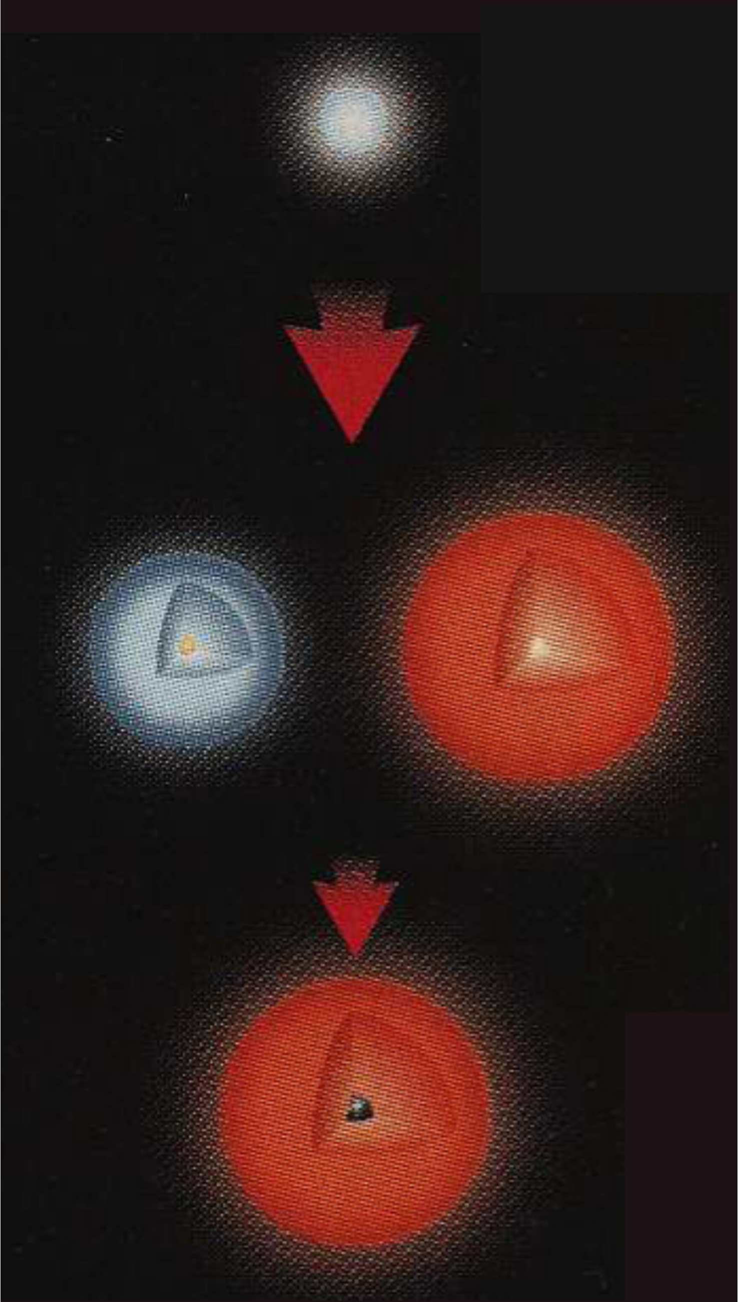
Helix Nebula
NGC 7293

HST ▪ ACS
NOAO 0.9m ▪ Mosaic I Camera

NASA, NOAO, ESA, The Hubble Helix Team,
M. Meixner (STScI), and T.A. Rector (NRAO)

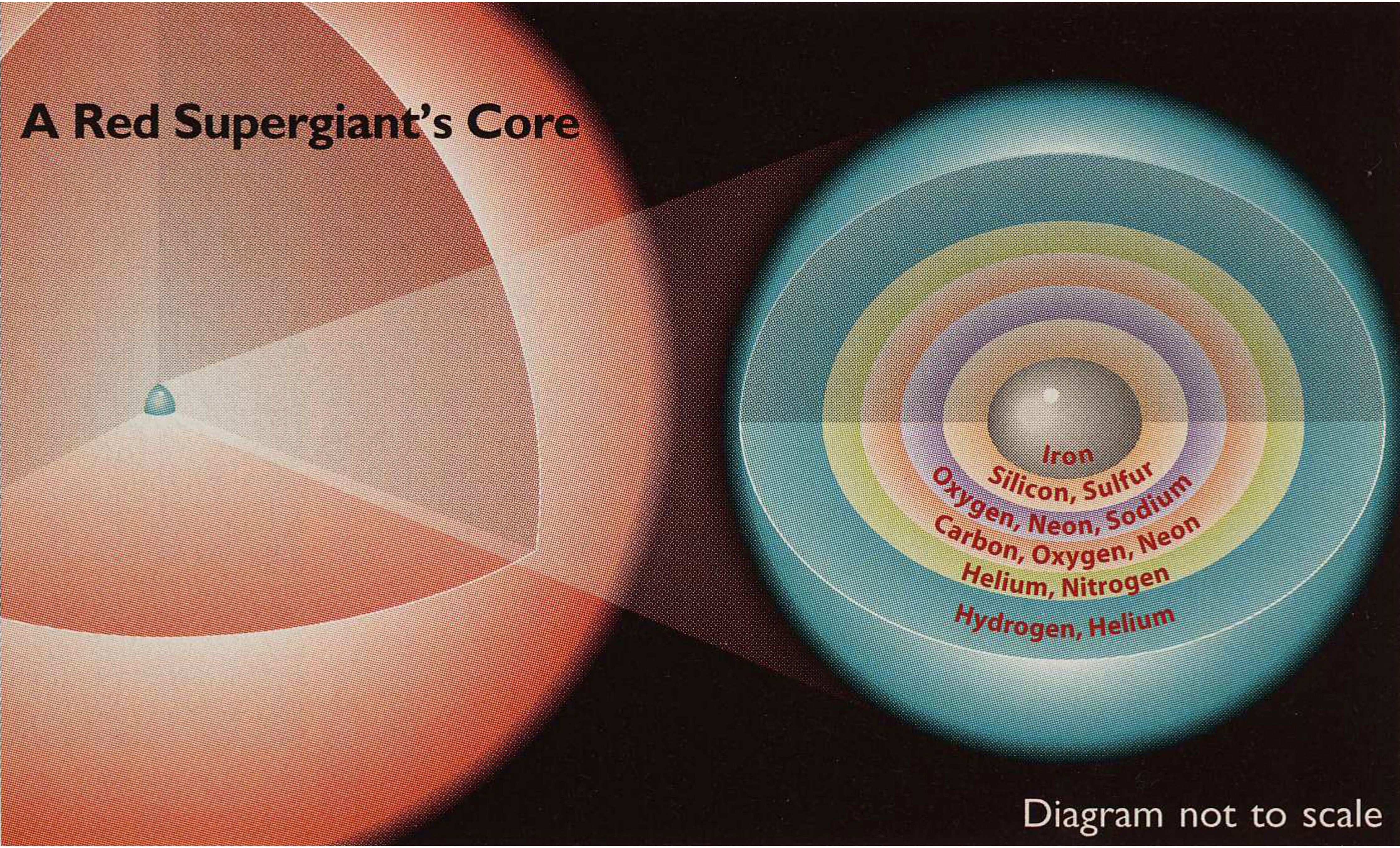
STScI-PRC03-11a


Life Cycle of a Sun-like Star
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
Luminosity (Suns)
Planetary nebula
Core helium flash begins . . .
Asymptotic branch giant
Red giant
. . . and ends
MAIN SEQUENCE
Main-sequence star
White dwarf
100,000
10,000
1,000
Surface temperature (degrees Kelvin)



Massive stars

- Mass between 10 and 50 $M_{\odot}$
- Short main-sequence phase
  → red or blue ***supergiant***
  with He core
- He burning → C-O core
- Core contraction → carbon burning
  → oxygen-neon core → neon burning
- Several cycles of
  contraction – heating – ignition
  → ***red supergiant with iron core***


A Red Supergiant's Core
Iron
Silicon, Sulfur
Oxygen, Neon, Sodium
Carbon, Oxygen, Neon
Helium, Nitrogen
Hydrogen, Helium
Diagram not to scale

Massive stars – core collapse

- When mass of iron core > 1.4 $M_\odot$
  → core collapses within fraction of second to size of small city
- Iron nuclei photodisintegrate → He nuclei → neutrons → ***neutron star***
- Matter above core ejected with high velocities → ***Type II supernova***
  − luminosity comparable to entire galaxy
- Chemical elements produced during explosion are returned to interstellar medium

Animation of supernova explosion dissolving into Chandra First Light Image, Cassiopeia A

http://chandra.harvard.edu/resources/animations/sn_to_casa_sm_web.mov

Galactic supernova remnants

3 pc

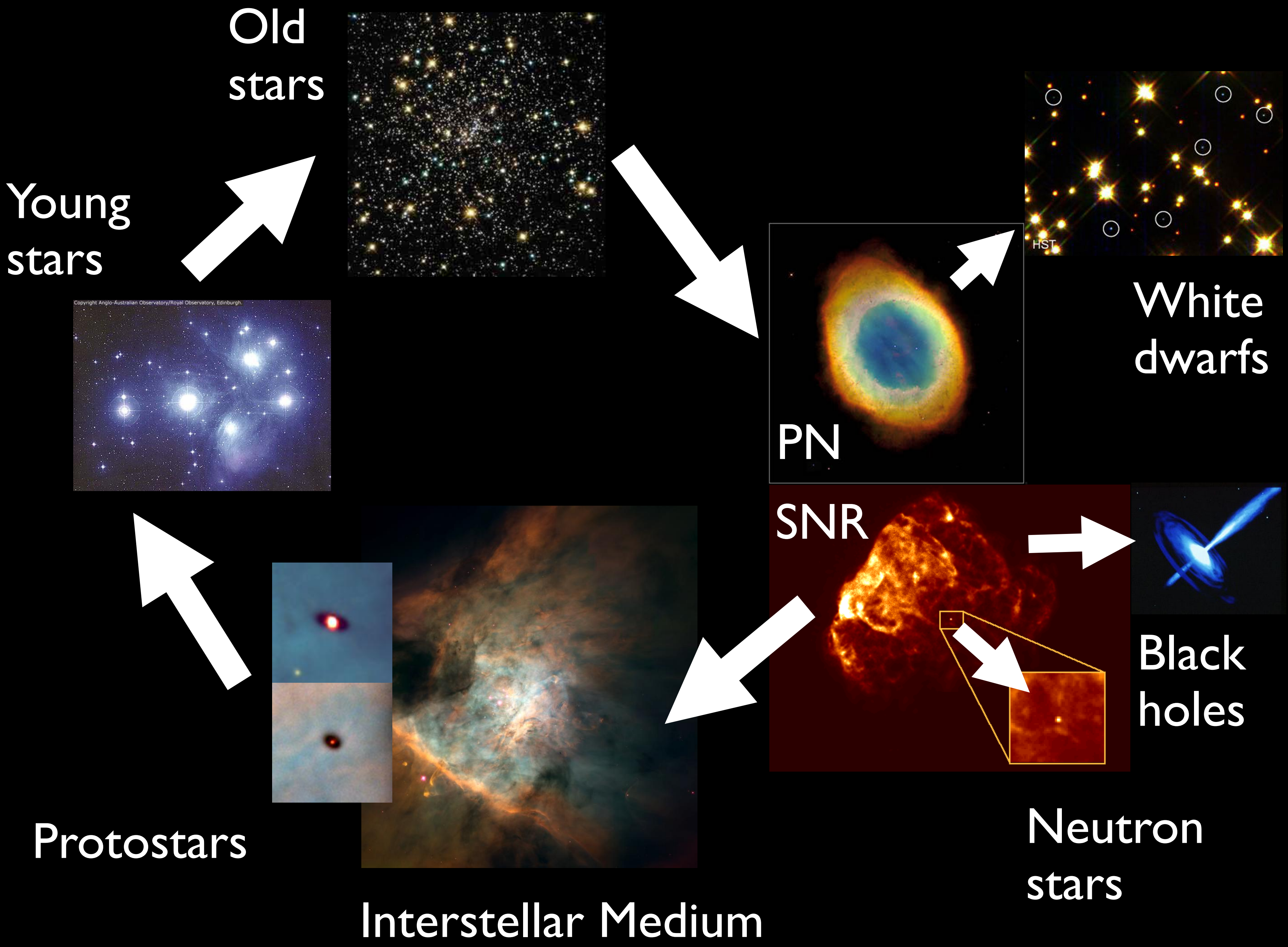
Crab, 1000 years ago

Credit: FORS Team, 8.2-meter VLT, ESO

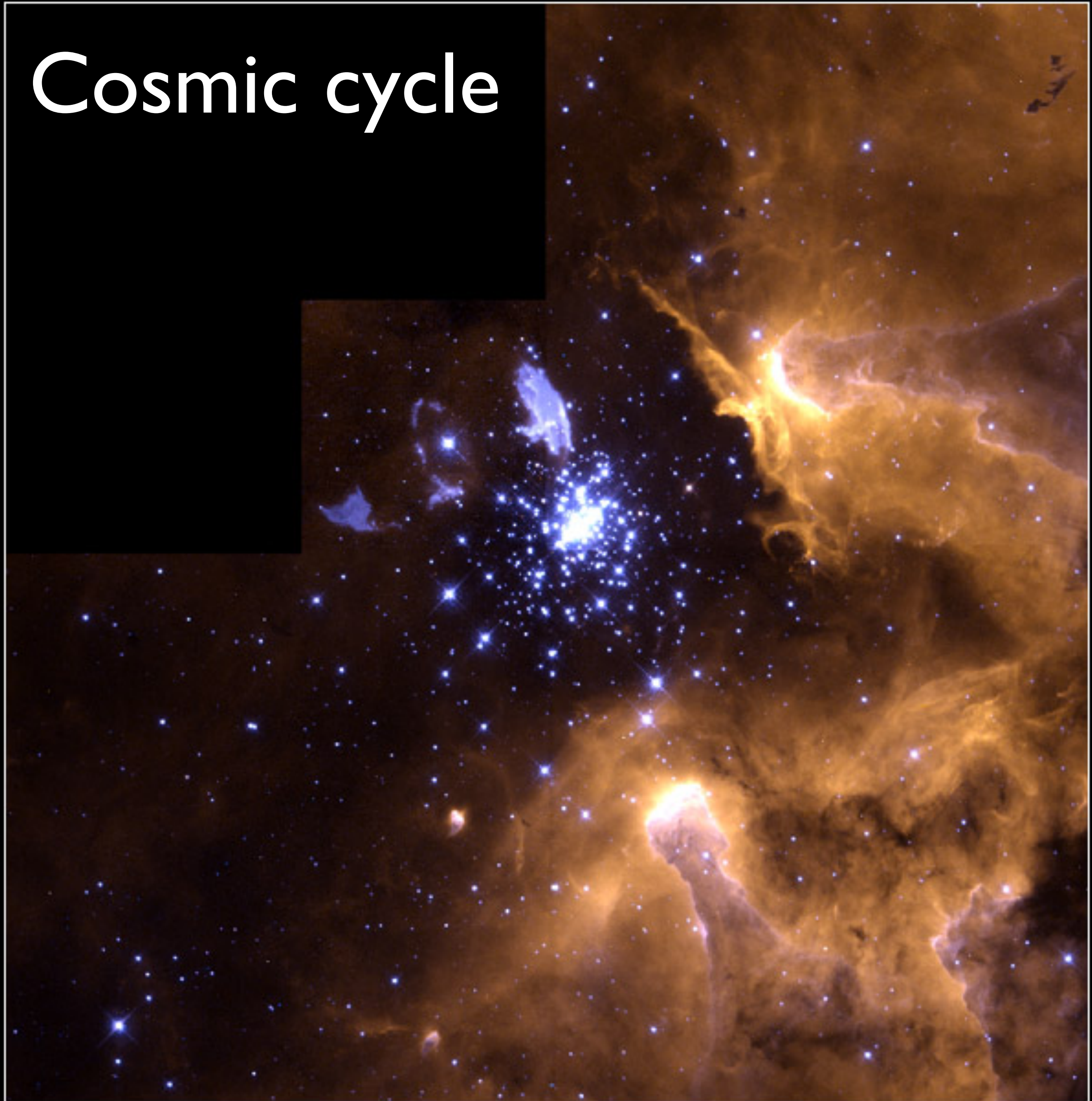
30 pc

Vela, 10000 years ago

Cosmic cycle



Cosmic cycle



NGC 3603 **HST** • WFPC2

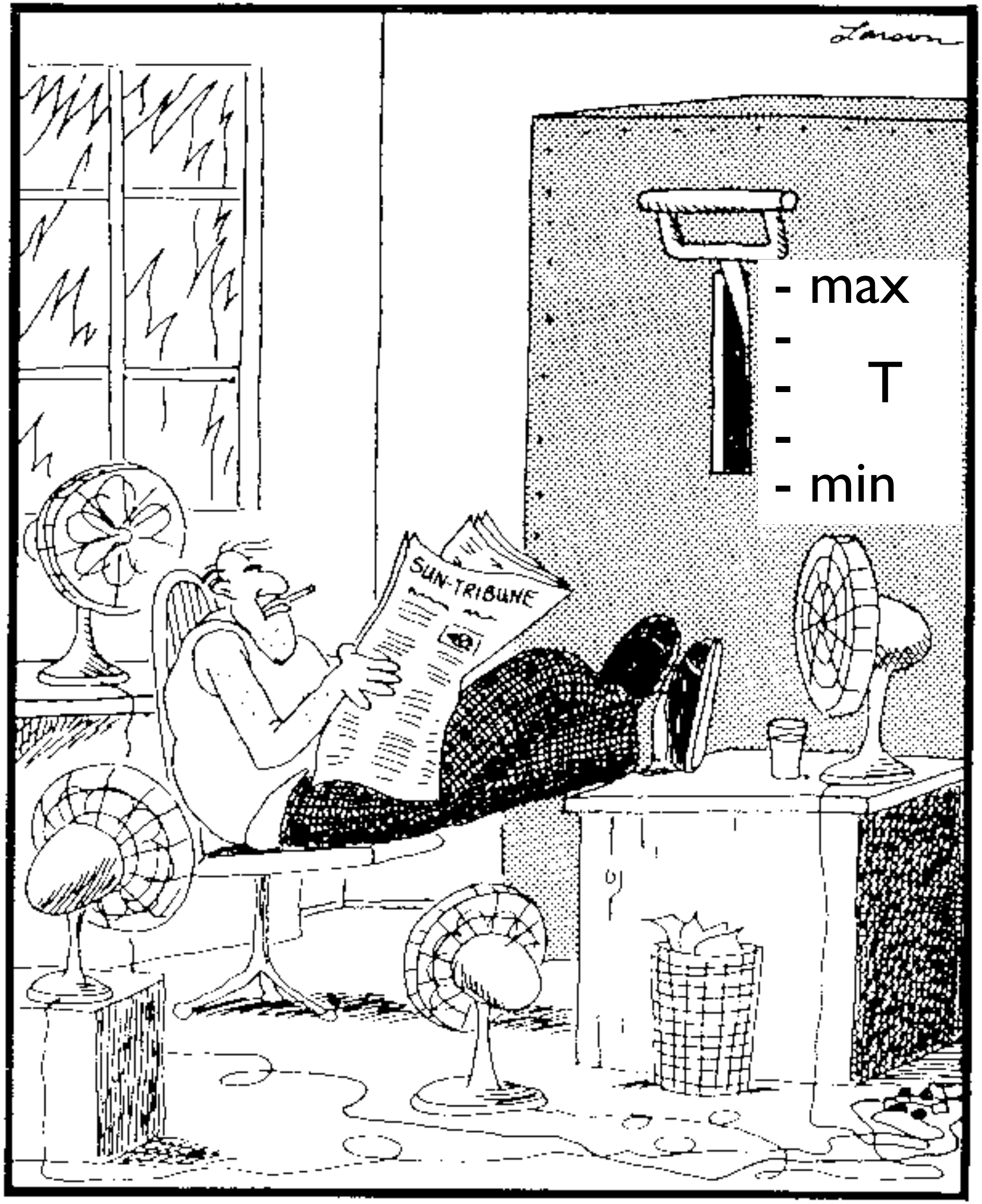
PRC99-20 • STScI OPO • June 1, 1999

Wolfgang Brandner (JPL/IPAC), Eva K. Grebel (Univ. Washington), You-Hua Chu (Univ. Illinois, Urbana-Champaign) and NASA

Physics of stellar evolution

- Equations of stellar structure and evolution
- Timescales of stellar evolution
- Evolutionary tracks

Inside the Sun?



Basic assumptions

- Star = radiating body of gas bound by self-gravity
- Radiation originates from internal energy sources
  - Nuclear energy from fusion reactions
  - Gravitational potential energy from contraction
- Star in empty space, not influenced by other stars
  - Average distance between stars much larger than average star diameter
  - Gravity and radiation decrease by factor 10^{-15} from one star to another
- Ignore deviations from spherical symmetry (e.g. rotation, magnetic field)

Basic assumptions

- Physical properties are uniform over spherical surface of radius *r*
- Local thermodynamic equilibrium (**LTE**)
  - Average properties of the gas in volume element at *r* are described by **local state variables**
  - Temperature T, density ρ, pressure P, internal energy u, energy release rate q, luminosity L, chemical composition
  - Changes of state variables are small over *mean free path* of particles and over *mean free time* between collisions between particles, including photons

Basic assumptions

- Initial **chemical composition** homogeneous as in Sun
  - **Number** abundances, from abundance analysis:
    $[M_i/H] = \log[n(M_i)/n(H)] - \log[n(M_i)_\odot/n(H)_\odot]$
  - **Mass** fractions: hydrogen → X, helium → Y,
    all metals (M) → **metallicity** Z, any species M_i → X_i

$$[\mathrm{M/H}] \approx [\mathrm{Fe/H}] \approx \log(Z/Z_\odot)$$

$$Z_\odot \approx 0.014$$

[Fe/H]	Z
–1.3	0.0007
–1.0	0.0014
0.0	0.014
+0.4	0.035

Model of a star

- ***Stellar structure*** =
  State variables as a function of *r* (distance from center)
- ***Stellar evolution*** =
  Change of structure with *t* (time)
- Derive ***equations for state variables*** as functions of *r* and *t* from
  - conservation laws
  - nuclear physics (nuclear reaction rates)
  - description of energy transport (by radiation and convection)
  - atomic physics (opacity)
  - equation of state

Conservation of mass and momentum

- $m(r)$ … mass contained in sphere with radius r
- dm … mass of shell between r and $r+dr$

$$dm = 4\pi r^2 \rho dr$$

- **Use *m* as independent variable**, because total stellar mass changes less during evolution than stellar radius
- **Conservation of momentum** → equation for **pressure**
- Nuclear reaction rates → equations for **mass fractions** of chemical species, X_i

Energy

- Conversation of energy and energy transport
  $\rightarrow$ equations for **luminosity** and **temperature**
- **Transport of energy** inside stars by
  1. Conduction (collisions between particles)
     - cores of evolved stars, white dwarfs, neutron stars
  2. **Convection**
     - occurs in case of intense nuclear burning or high opacity
     - important effects: energy transport and **mixing**
  3. **Radiation**
     - diffusion approximation

Radiative transport

- **Mean free path** of photons:

$$\ell_{\rm ph} = \frac{1}{\kappa\rho}$$

κ … radiative cross-section per unit mass (absorption coefficient, **opacity**)

typical values: $\kappa = 1\ \mathrm{cm^2\,g^{-1}}$, $\rho = 1\ \mathrm{g\,cm^{-3}} \rightarrow \ell_{\rm ph} = 1$ cm

- Compared to stellar radius, $\ell_{\rm ph}$ is small $\rightarrow$ can treat radiative transport as a **diffusion process** **= random walk of photons**

- How long is the travel time $t_{\rm ph}$ of photons from the interior to the surface of the Sun? Notes: distance travelled ≈ step size times square root of number of steps; detailed calculation in Mitalas & Sills 1992, ApJ 401, 759

Radiative transport

- How long is the travel time t_{ph} of photons from the interior to the surface of the Sun?

➢ Number of steps after which distance $R_\odot$ is reached:

$$N = \left(\frac{R_\odot}{\ell_{\rm ph}}\right)^2$$

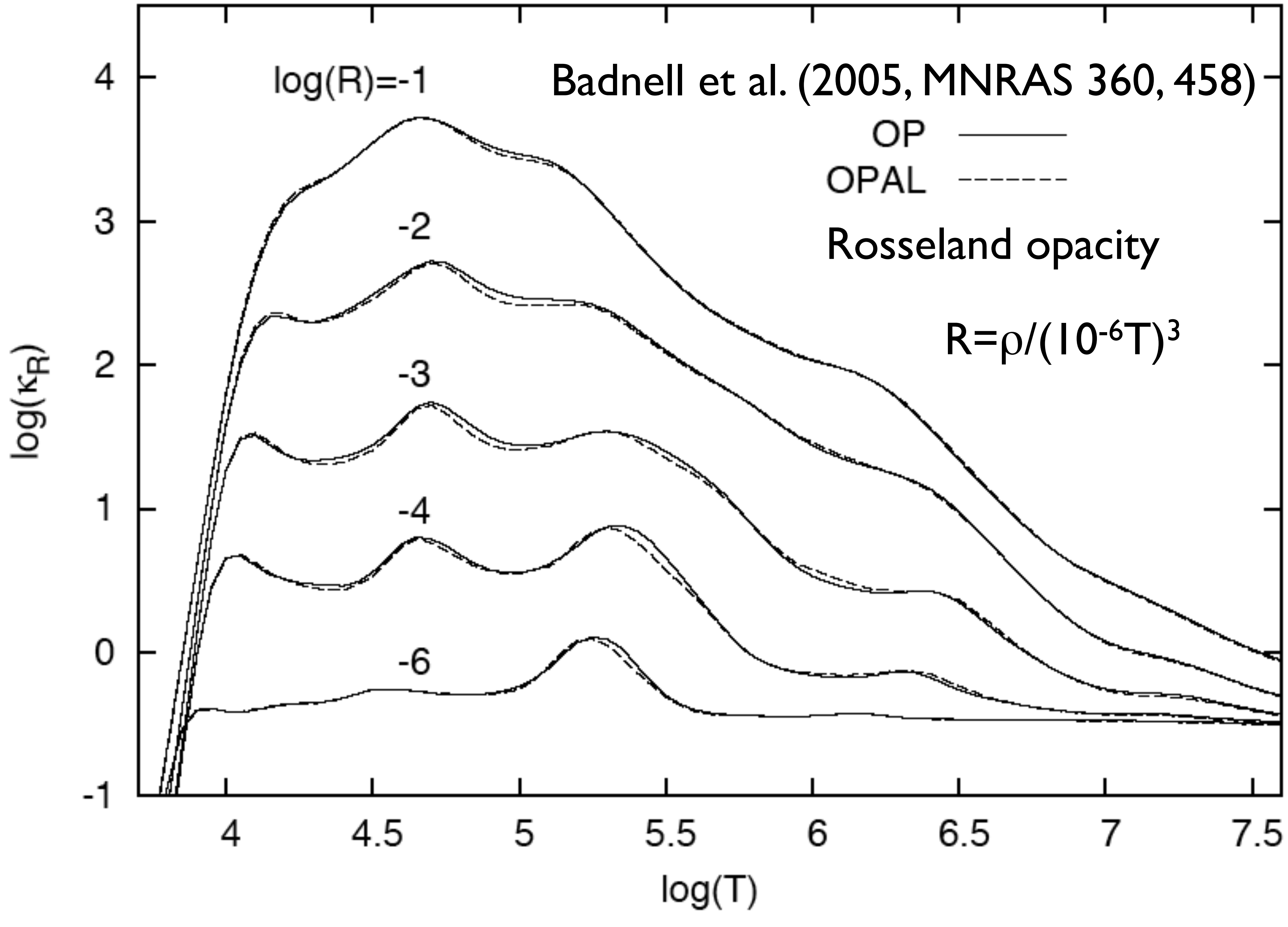
$$\Rightarrow t_{ph} = \frac{N\ell_{\rm ph}}{c} \approx \frac{(7\cdot 10^{10})^2}{3\cdot 10^{10}\cdot 3\cdot 10^7} \text{ [cgs]} \approx 5000 \text{ yr}$$

ℓ_{ph}= 1 cm

Mitalas & Sills 1992: $<\ell_{ph}>$= 0.09 cm, t_{ph}=170000 yr

Opacity from atomic physics

- Absorption coefficient calculated for all possible interactions between particles and photons
- Tabulated for different combinations of T, ρ and X_i
- Or approximated by power law: $$\kappa = \kappa_0 \rho^a T^b$$
- Most important interactions:
  electron scattering – a=0, b=0; free-free absorption – a=1, b=–7/2
- Opacity tables
  - OPAL (Rogers and Iglesias), also equation of state tables:
    http://rdc.llnl.gov/opal.html
  - Opacity Project (International collaboration)
    http://cdsweb.u-strasbg.fr/topbase/TheOP.html



Badnell et al. (2005, MNRAS 360, 458)
OP
OPAL
Rosseland opacity
$R=\rho/(10^{-6}T)^3$
log(R)=-1
-2
-3
-4
-6
$\log(\kappa_R)$
log(T)
4
3
2
1
0
-1
4
4.5
5
5.5
6
6.5
7
7.5

Equations of stellar structure and evolution

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\ddot{r}$$

$$\frac{\partial L}{\partial m} = q - \dot{u} - P\left(\frac{1}{\rho}\right)^{\cdot}$$

$$\frac{\partial T}{\partial m} = -\frac{3}{16\sigma}\frac{\kappa}{T^3}\frac{L}{(4\pi r^2)^2}$$

$$\frac{\partial X_i}{\partial t} = f_i(\rho, T, \vec{X})$$

Equation of state

$$\kappa = \kappa_0 \rho^a T^b, q = q_0 \rho^m T^n, P = f(\rho, T, X_i)$$

Equations of stellar structure and evolution

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \cancel{\frac{1}{4\pi r^2}\ddot{r}}$$

$$\frac{\partial L}{\partial m} = q \cancel{- \dot{u}} \cancel{- P\left(\frac{1}{\rho}\right)^{\cdot}}$$

$$\frac{\partial T}{\partial m} = -\frac{3}{16\sigma}\frac{\kappa}{T^3}\frac{L}{(4\pi r^2)^2}$$

$$\cancel{\frac{\partial X_i}{\partial t}} = f_i(\rho, T, \vec{X})$$

Equilibrium → stellar structure equations

Equation of state

$$\kappa = \kappa_0 \rho^a T^b, q = q_0 \rho^m T^n, P = f(\rho, T, X_i)$$

Characteristic timescales of stellar evolution

- Equations of stellar evolution describe changes for three different properties on different timescales
- Structural changes → **dynamical timescale**

$$t_{\rm dyn} = \frac{R}{\dot{R}} \approx \frac{R}{v_{\rm ff}} = \left(\frac{R^3}{2GM}\right)^{1/2} \approx \left(\frac{1}{G\bar{\rho}}\right)^{1/2}$$

  - ***Sun***: $t_{\rm dyn} \approx$ 15 minutes, ***red giants*** ($100 R_\odot$): 2 weeks, ***white dwarfs*** ($R_\odot/50$): 3 seconds
  - $t_{\rm dyn}$ = typical timescale on which star reacts to perturbation of hydrostatic equilibrium

Characteristic timescales of stellar evolution

- Change of internal energy U_{tot} → **thermal timescale**
  - Virial theorem: $|U_{\text{tot}}| = \frac{1}{2}\Omega$

    Ω … total gravitational potential energy

    Change in U_{tot} = luminosity

$$t_{\text{th}} = \frac{U_{\text{tot}}}{L} \approx \frac{GM^2}{2RL}$$

  - ***Sun***: t_{th} ≈ 15 million years
  - Much longer than t_{dyn}, but much shorter than stellar lifetime → stars mostly in thermal equilibrium
  - t_{th} = time a star could maintain constant luminosity by contraction = *Kelvin-Helmholtz timescale*

Characteristic timescales of stellar evolution

- Change in composition and "nuclear potential energy" E_n → **nuclear timescale**
  - E_n given by *mass excess of hydrogen per unit mass* times *hydrogen mass fraction* times *stellar mass*
  - Change in E_n = nuclear luminosity L_{nuc} = L in thermal equ.

$$t_{\mathrm{nuc}} \approx \frac{7\mathrm{MeV}/m_{\mathrm{H}} X M}{L}$$

  - ***Sun***: $t_{nuc} \approx$ 100 billion years, much larger than its age
  - Stars don't consume all available nuclear energy
  - Only fraction of stellar mass changes composition

Characteristic timescales of stellar evolution

$$\boxed{t_{\text{dyn}} \ll t_{\text{th}} \ll t_{\text{nuc}}}$$

$\rightarrow$ rates of nuclear processes determine stellar evolution
$\rightarrow$ can decouple equation for chemical composition from the others

1. Solve stellar structure equations for given composition
2. Apply time step and determine new composition

Solving for stellar structure

- **Four** differential equations with **boundary conditions** at
  - center of star ($m = 0$): $r = 0, L = 0$
  - surface of star ($m = M$): fit interior solution to a stellar atmosphere model
- Three "material functions" (for ρ, κ, q)
- ***Input*** parameters:
  mass $\boldsymbol{M}$, chemical composition $\mathbf{X}(\boldsymbol{t}), \mathbf{Y}(\boldsymbol{t}), \mathbf{Z}(\boldsymbol{t})$
- ***Output***: $r(m), P(m), L(m), T(m), \rho(m), \kappa(m), q(m)$, for each time t, in particular $\boldsymbol{T_{\mathrm{eff}}}, \boldsymbol{L}, \boldsymbol{R}, \boldsymbol{\rho_c}, \boldsymbol{P_c}$
- Equations are highly non-linear and coupled → have to be solved with numerical methods

Examples of stellar evolution models and codes

- Geneva grids of stellar evolution models

  database of models for masses between 0.8 and 120 solar masses, pre-main-sequence, and from main sequence up to carbon burning

  http://obswww.unige.ch/Recherche/evol/Geneva-grids-of-stellar-evolution

- Padova database of stellar evolutionary tracks

  http://stev.oapd.inaf.it/

- Dartmouth Stellar Evolution Web Server

  Java applet for calculation of models between 0.5 and 5 solar masses, from main sequence to core helium burning

  http://stellar.dartmouth.edu

- CESAM (Nice)

  Code d'Évolution Stellaire Adaptatif et Modulaire

  http://www-n.oca.eu/cesam/

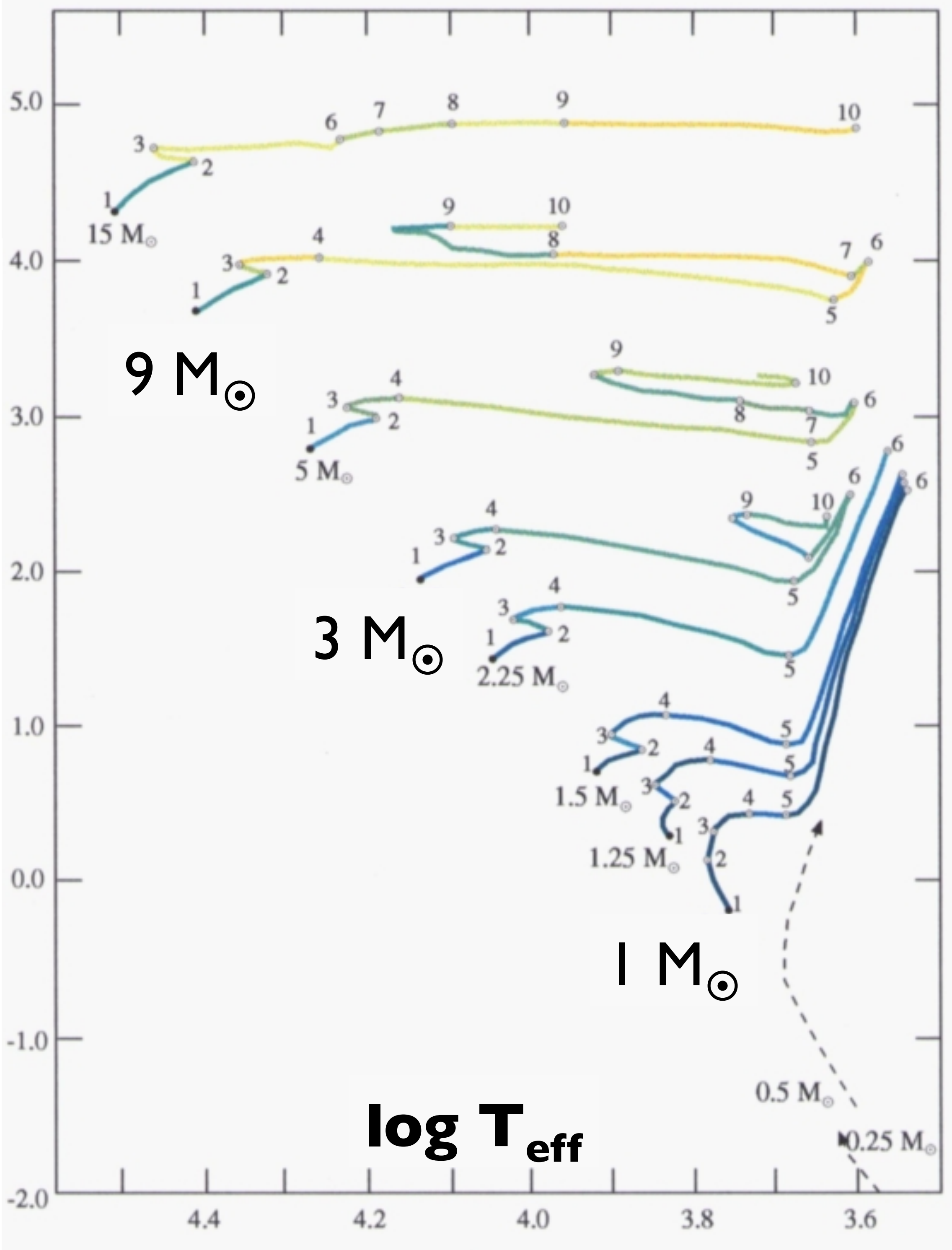
Evolutionary tracks in theoretical HR diagram

- Main sequence

  1→2: Core H burning

  2→3: X_{center} 0.05→0

- Shell H burning (3→5)
  - intermediate masses (≥2$M_\odot$): fast → "Hertzsprung gap"
  - low masses: slow
- Red giant branch (RGB, 5→6)
- Helium ignition in core at 6



Summary of Part 2

- Assuming stars to be radiating gas spheres and applying basic physics of gas and radiation leads to **differential equations for stellar structure** variables.
- Estimation of **characteristic timescales** allows to decouple equations and solve for changes in space and time separately.
- Stellar evolution models predict **surface properties of stars of different mass and age**, which can be compared to observations.