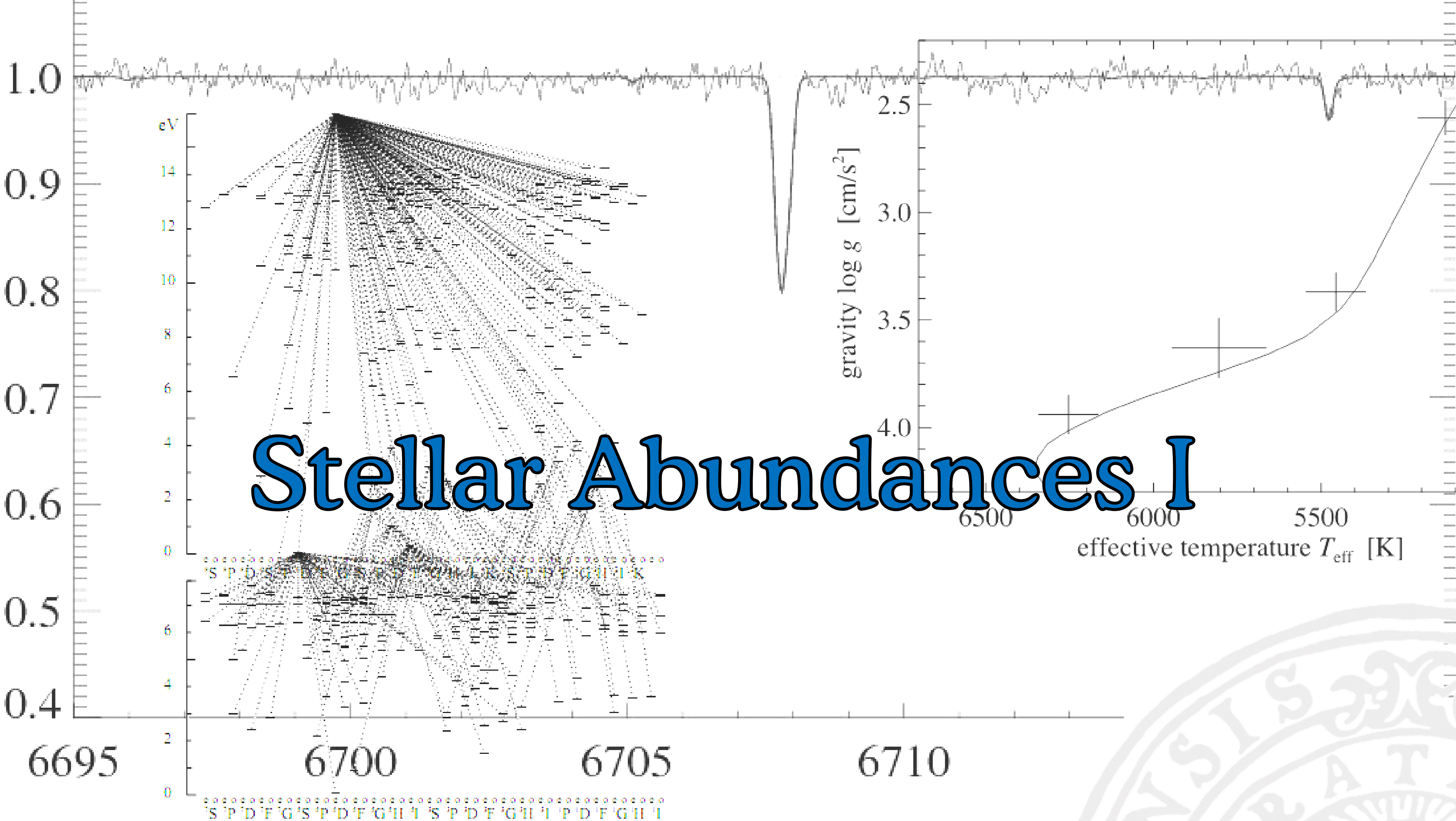

Stellar Abundances I

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Observational Stellar Astrophysics in the Era of Gaia and Kepler Space Missions

Moletai, 2012-07-31

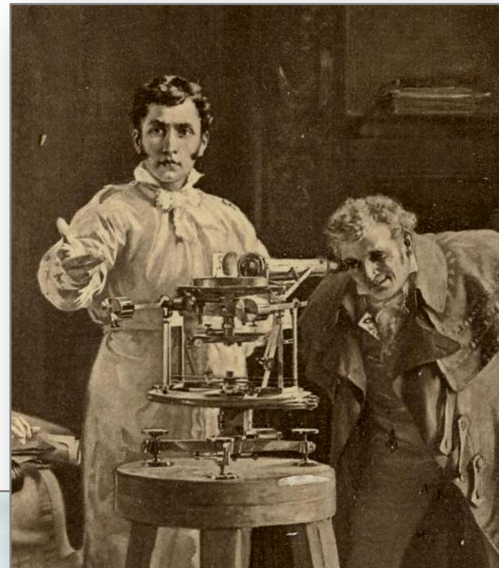
What will be covered

I. **Theoretical background** (introductory)
   observables, radiative transfer, opacities and line formation
   model atmosphere output
   how lines depend on T_{eff}, $\log g$, $\log \varepsilon(\mathrm{X})$ etc.

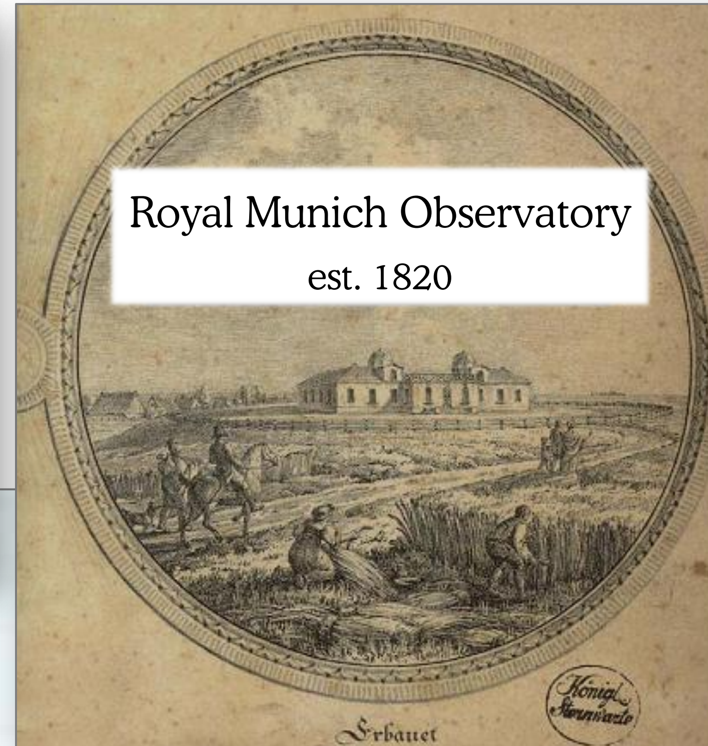
II. **Methods of stellar-parameter and chemical-abundance determination** (Thursday)
   fundamental stellar parameters
   photometry (in a nutshell)
   spectroscopy (a practical selection)

III. **Exercise** (Friday afternoon)

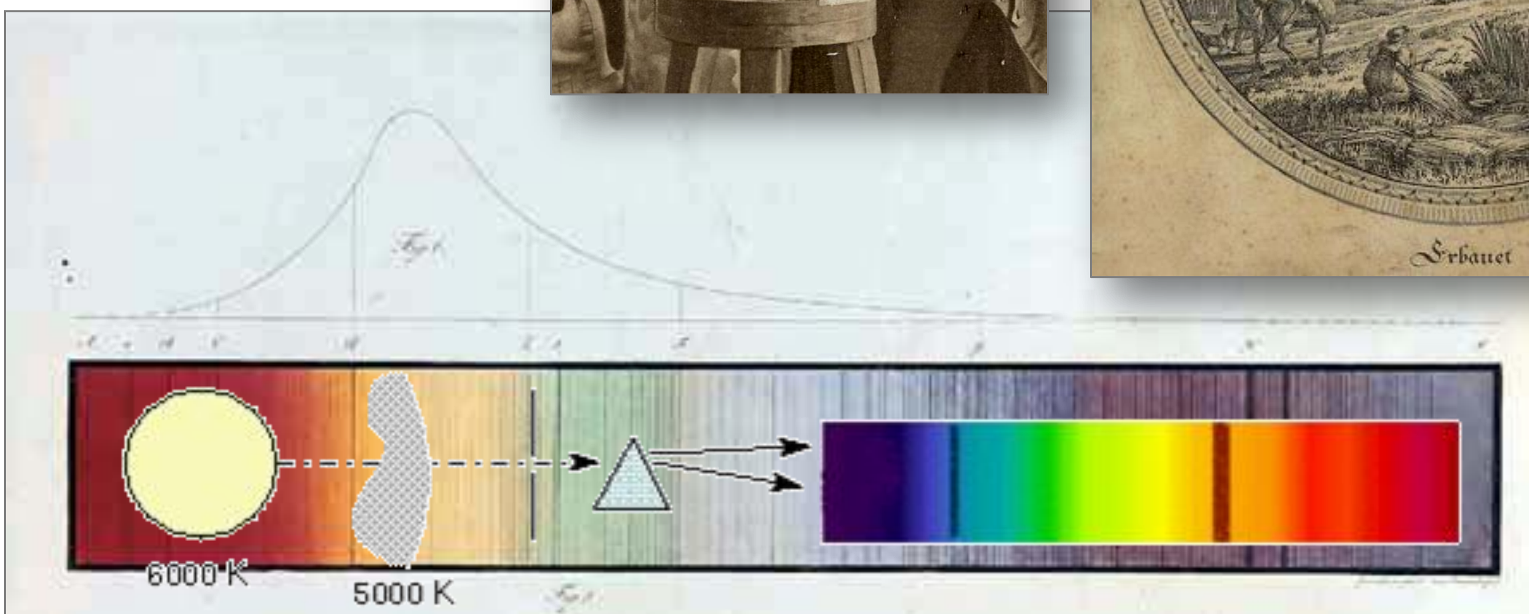
Stellar spectroscopy in the early days



Joseph von Fraunhofer
(1787–1826)



Royal Munich Observatory
est. 1820



Stellar spectroscopy today

observation vs. theory

NOT

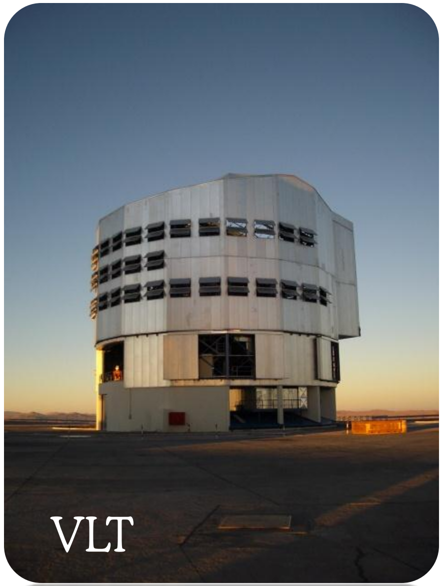
telescope

FIES

spectrograph

E2V 42-40

CCD



$$\frac{d\mathcal{I}_\nu}{d\tau_\nu} = -\mathcal{I}_\nu + S_\nu$$

concepts

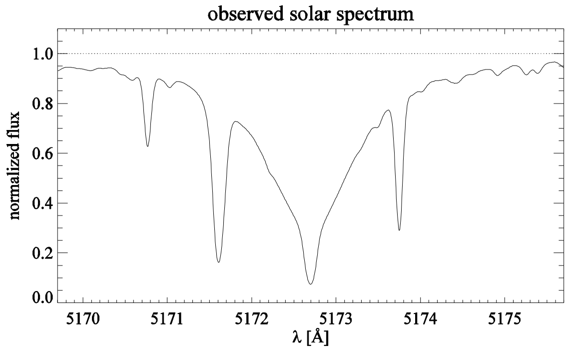
$$\frac{d}{dx}\mathcal{F} = 0$$

LTE

approximations

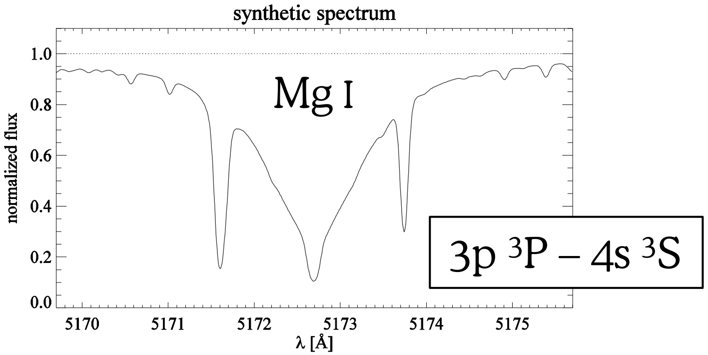
MLT

numerical model

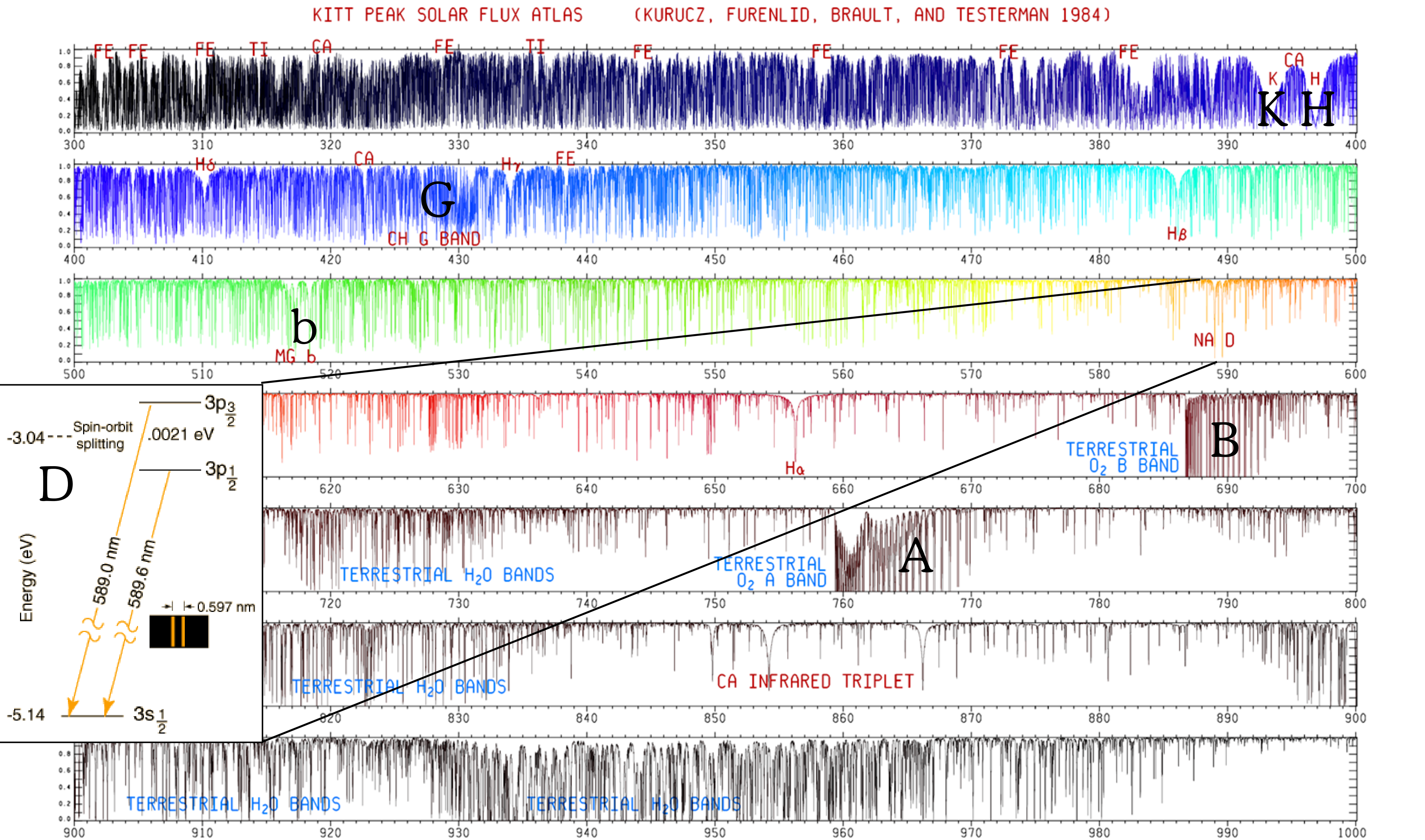


comparison

to constrain thermodynamic variables and abundances



The Solar spectrum



What are stellar parameters?

There are different ways of looking at what defines a stars:

stellar-structure view $M, \mathcal{L}, X, Y, Z, R, \upsilon_{\mathrm{rot}}, t, \ldots$

stellar-atmosphere view $\mathcal{F}_\nu, T_{\mathrm{eff}}, \log g, [\mathrm{X_i}/\mathrm{H}], \upsilon_{\mathrm{rot}} \sin i, \ldots$

$\log (\mathrm{G}\, M / R^2)$

While the prior is (often) more fundamental, the latter is more directly related to observations (photospheres!) and generally speaking more applicable. In this lecture, I will follow the latter view.

Linking input to output

Chemical abundances

Cosmic evolution

Output

Stellar ages

Nucleogenesis

Stellar parameters

Model atmospheres

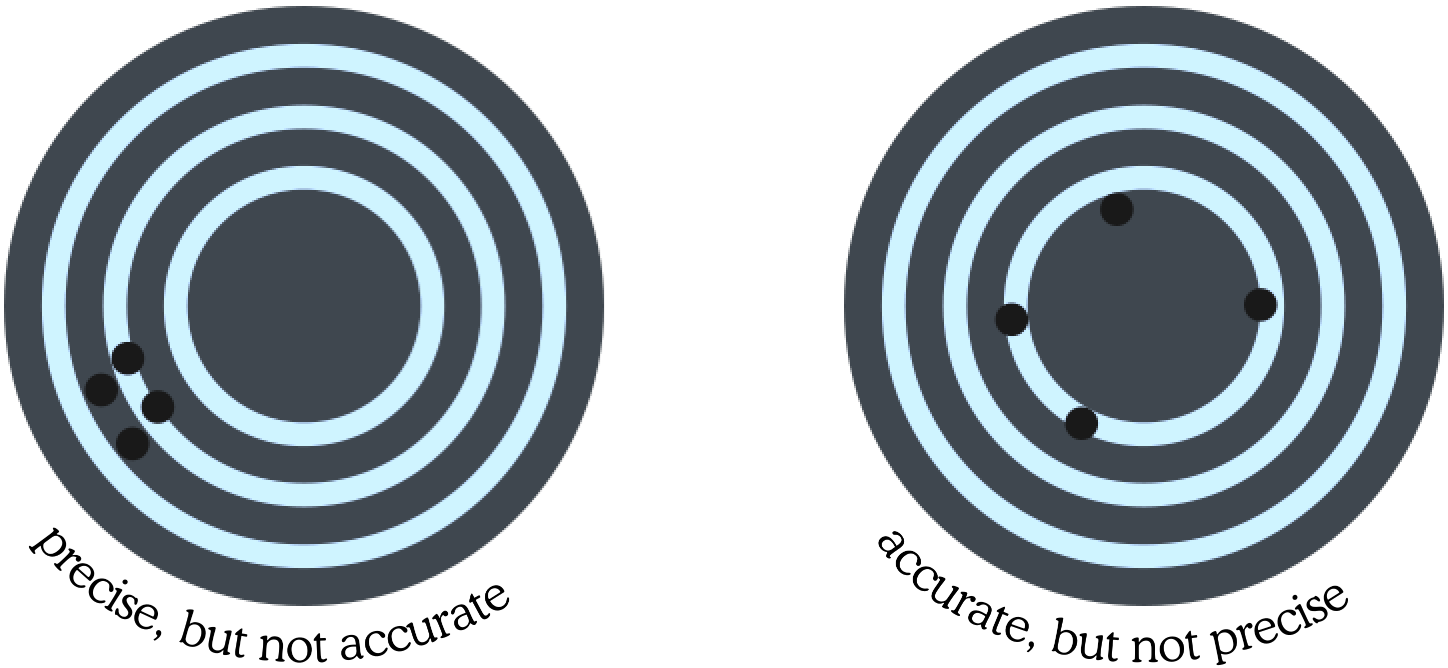
Stellar evolution

Input

Observations

Chemical evolution model

Precision *vs.* accuracy



NB: Some projects may require high precision *and* accuracy, while for others it will suffice to reach some level of precision.

Stellar atmosphere: a definition

descriptive: the layers of a star from which we receive photons = the layers we can see

physical: $0 \leq \tau_\nu \leq 10$

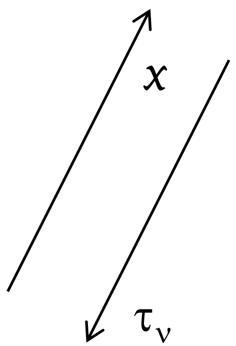
where $\tau_\nu = (-)\int_0^L \kappa_\nu \, \rho \, dx$ is the **optical height**,

x measures the geometrical path [cm],

ρ is the mass density [g cm^{-3}],

κ_ν is the mass absorption coefficient [cm^2 g^{-1}] and

L is the path length (see Gray, ch. 5, p. 113)



optical *depth*

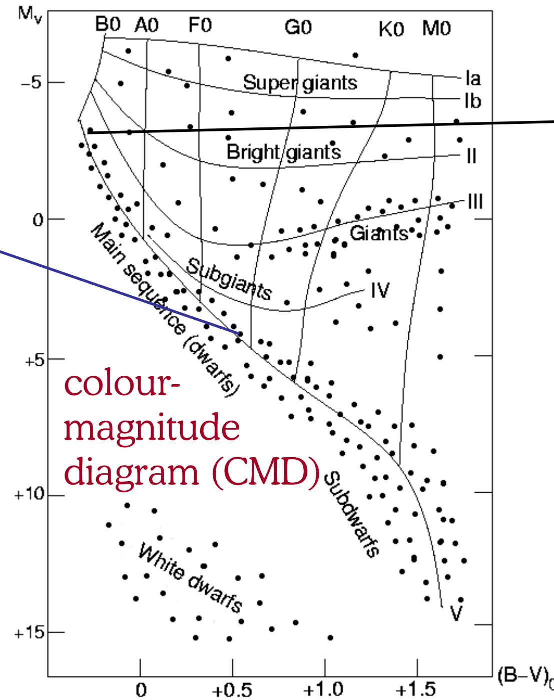
simple extinction law: $\mathcal{I}(\nu) = \mathcal{I}_0(\nu) \exp(-\tau_\nu)$

Stellar atmospheres: typical figures

The Sun

$M = 2 \times 10^{33}$ g $= \mathrm{M}_\odot$
$R = 7 \times 10^{10}$ cm $= \mathrm{R}_\odot$
$\mathcal{L} = 4 \times 10^{33}$ erg/s $= \mathrm{L}_\odot$

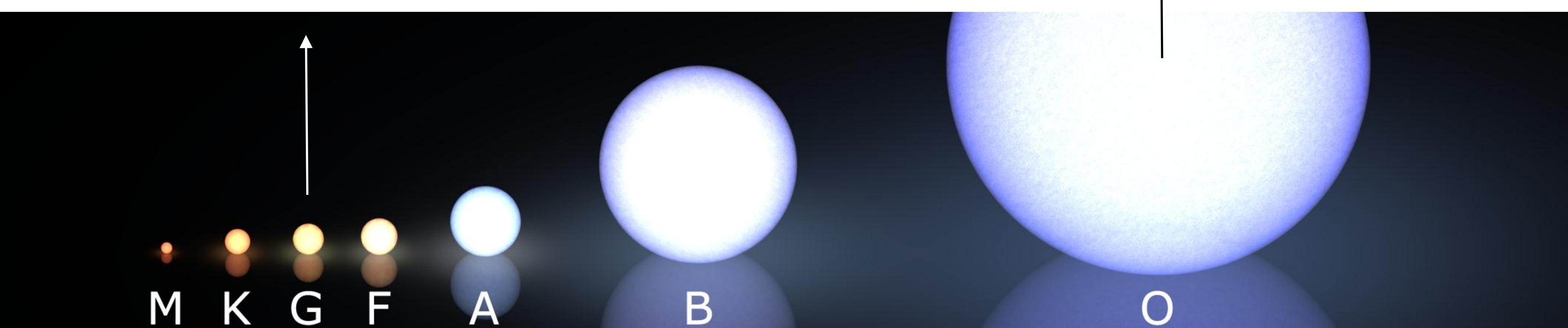
photosphere:
$\Delta R \approx 200$ km $< 10^{-3}\ \mathrm{R}_\odot$
$n \approx 10^{15}\ \mathrm{cm}^{-3}$
$T \approx 6000$ K



an O star

$M \approx 50\ \mathrm{M}_\odot$
$R \approx 20\ \mathrm{R}_\odot$
$\mathcal{L} \approx 10^6\ \mathrm{L}_\odot\ (\propto M^3)$

photosphere:
$\Delta R \approx 0.1\ \mathrm{R}_\odot$
$n \approx 10^{14}\ \mathrm{cm}^{-3}$
$T \approx 40\,000$ K



wikipedia: stellar classification

Abundance nomenclature

Mass fractions: let X, Y, Z denote the mass-weighted abundances of H, He and all other elements ("metals"), respectively, normalized to unity ($X + Y + Z = 1$).

example: $X = 0.715$, $Y = 0.270$, $Z = 0.014$ for the proto-Sun

The 12 scale: $\log \varepsilon(\mathrm{X}) = \log (n_\mathrm{X} / n_\mathrm{H}) + 12$ $(\log \varepsilon(\mathrm{H}) \equiv 12)$

example: $\log \varepsilon(\mathrm{O})_\odot \approx 8.7$ dex, i.e., oxygen, the most abundant metal, is 2000 times less abundant than H in the Sun (the exact value is still hotly debated!)

Square-bracket scale: $[\mathrm{X/H}] = \log (n_\mathrm{X} / n_\mathrm{H})_\star - \log (n_\mathrm{X} / n_\mathrm{H})_\odot$

example: $[\mathrm{Fe/H}]_{\mathrm{HE0107-5240}} = -5.3$ dex, i.e., this star has an iron abundance a factor of 200 000 below the Sun (Christlieb *et al.* 2002)

Intensity and flux

The Sun is one of the few stars whose surface we can resolve , measure the so-called specific intensity

$$\mathcal{I}_\nu = dE_\nu / \cos\vartheta \, dA \, d\Omega \, dt \, d\nu \qquad [J / m^2 \, rad \, s \, Hz]$$

Usually, we measure stellar fluxes

$$\mathcal{F}_\nu = \int dE_\nu / dA \, dt \, d\nu \qquad [J / m^2 \, s \, Hz]$$

Clearly, the **flux** $\mathcal{F}_\nu = \int \mathcal{I}_\nu \cos\vartheta \, d\Omega$ and it **measures** the **anisotropy of the radiation field**.

Example: the Solar flux above the Earth's atmosphere

$$\mathcal{F}(\odot) = 1.36 \text{ kW} / m^2$$

Flux constancy and luminosity

Stellar atmospheres are much too cool and tenuous to fuse nuclei.
⇒ the energy coming from the stellar core is merely transported through the atmosphere, either by radiation or convection.

$\mathcal{F}(x)$ = energy / unit area / unit time [J m^{-2} s^{-1}] = [W m^{-2}]

$$\mathrm{d}\,\mathcal{F}(x)\,/\,\mathrm{d}x = 0 \qquad \text{(generally: } \nabla\,\mathcal{F} = 0\text{)}$$

The spectrum of $\mathcal{F}$ (i.e. $\mathcal{F}_\nu$) will change with r, but not the integral value.

If $\mathcal{F}_{\mathrm{rad}} \gg \mathcal{F}_{\mathrm{conv}}$, then one speaks of **radiative equilibrium**.

(Karl Schwarzschild 1873–1916)

The total energy output of a star is called its **luminosity**

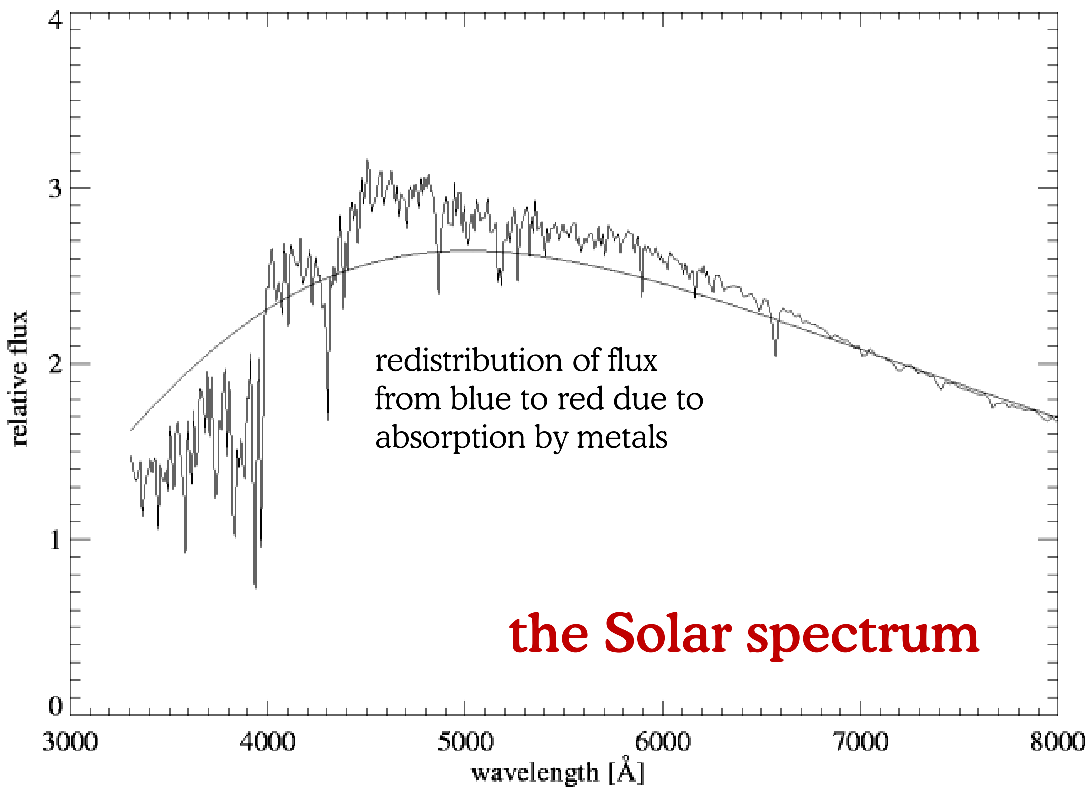
$$\mathcal{L} = 4\pi\, R^2\, \mathcal{F}(R)$$

Stellar spectra

Luckily, **stars** (and other celestial bodies) are not in thermodynamic equilibrium (TE) and **do not shine like blackbodies**.

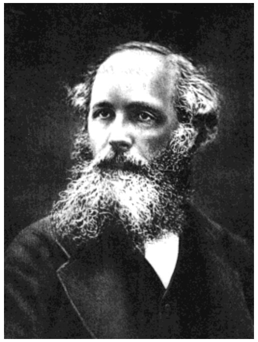
(Astronomy would be the dullest of all sciences!)

In constrast to $\mathcal{B}_\lambda$, $\mathcal{I}_\lambda$ depends on plasma properties and the viewing angle. **One cannot use TE to describe starlight.**



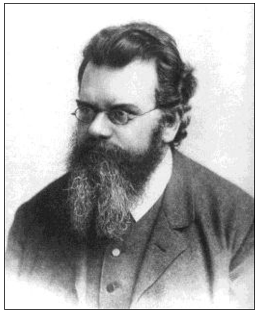
TE statistics

Particle velocities are assumed to be Maxwellian:



$$\frac{n(v)}{n_{\rm tot}}{\rm d}v = (\frac{m}{2\pi kT})^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}}{\rm d}v$$

Excitation follows the Boltzmann distribution:



$$\frac{n_u}{n_{\rm tot}} = \frac{g_u}{u(T)} e^{-\frac{\chi_u}{kT}}$$

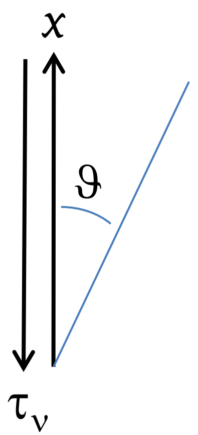
Ionization can be computed via the Saha equation:

$$\frac{n_{\rm II}}{n_{\rm I}} P_e = \frac{(2\pi m_e)^{3/2} kT^{5/2}}{h^3} \frac{2u_{\rm II}(T)}{u_{\rm I}(T)} e^{-\frac{I}{kT}}$$

In **local thermodynamic equilibrium** (LTE), these are applied *locally*.

The basics of radiative transfer

When the stellar photons interact with the stellar-atmosphere matter, photons can be absorbed and re-emitted. This is the basic message of the **radiative transfer equation**.



$$d\,\mathcal{I}_\nu = -\,\kappa_\nu\,\rho\,\mathcal{I}_\nu\,dx + j_\nu\,\rho\,dx$$ or

j_ν: emission coefficient

$$\cos\vartheta\; d\,\mathcal{I}_\nu \,/\, d\tau_\nu = +\,\mathcal{I}_\nu - \mathcal{S}_\nu$$ with

$$\mathcal{S}_\nu = j_\nu / \kappa_\nu$$ the **source function**

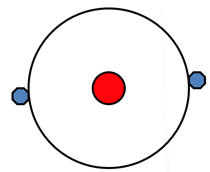
In LTE, $\mathcal{S}_\nu = \mathcal{B}_\nu$ the **Planck function**

$\mathcal{B}_\nu$ has a number of wonderful properties: it does not depend on material properties (only T) and increases monotonically with increasing T for all ν.

The integral $\int \mathcal{B} \cos\vartheta \; d\Omega$ yields σT^4 (Stefan-Boltzmann law).

Similarly, the **effective temperature T_{eff}** is defined: $\int \mathcal{F}_\nu \, d\nu = \sigma T_{\mathrm{eff}}^{\;4}$

Opacities



Continuous opacity

Caused by *bf* or *ff* transitions

In the optical and near-IR of cool stars, H^- ($I = 0.75$ eV) dominates:

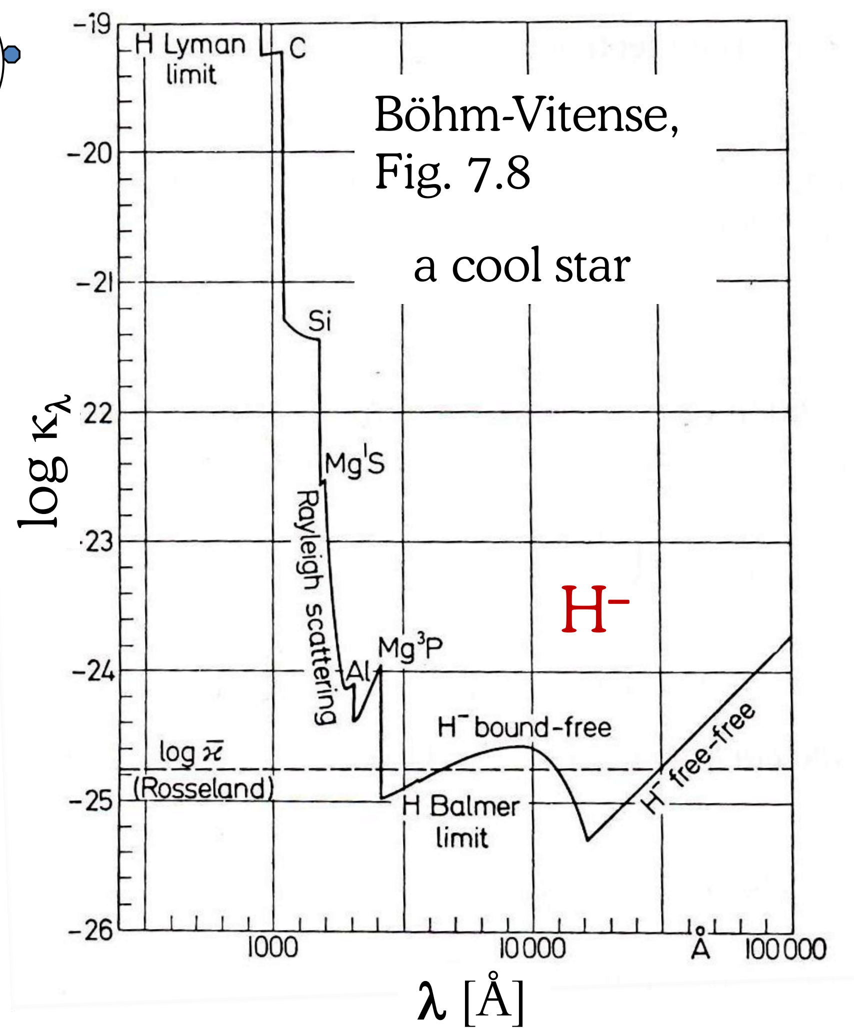
$$\kappa_\nu(H^-_{bf}) = \text{const.}\ T^{-5/2} P_e \exp(0.75/kT)$$

NB: There is only 1 H^- per 10^8 H atoms in the Solar photosphere.

Line opacity *(all the lines you see!)*

Caused by *bb* transitions

Need to know $\log gf$, damping and assume an abundance



Model atmosphere output

A 1D model atmosphere is a tabulation of various quantities as a function of (optical) depth:

- ***T*** (temperature)
- $\boldsymbol{P_g}$ (gas pressure)
- $\boldsymbol{P_e}$ (electron pressure)
- $\mathcal{F}_\nu$ (esp. *surface* flux) etc.

as computed under certain input assumptions:

T_{eff} (effective temperature)

log *g* (surface gravity)

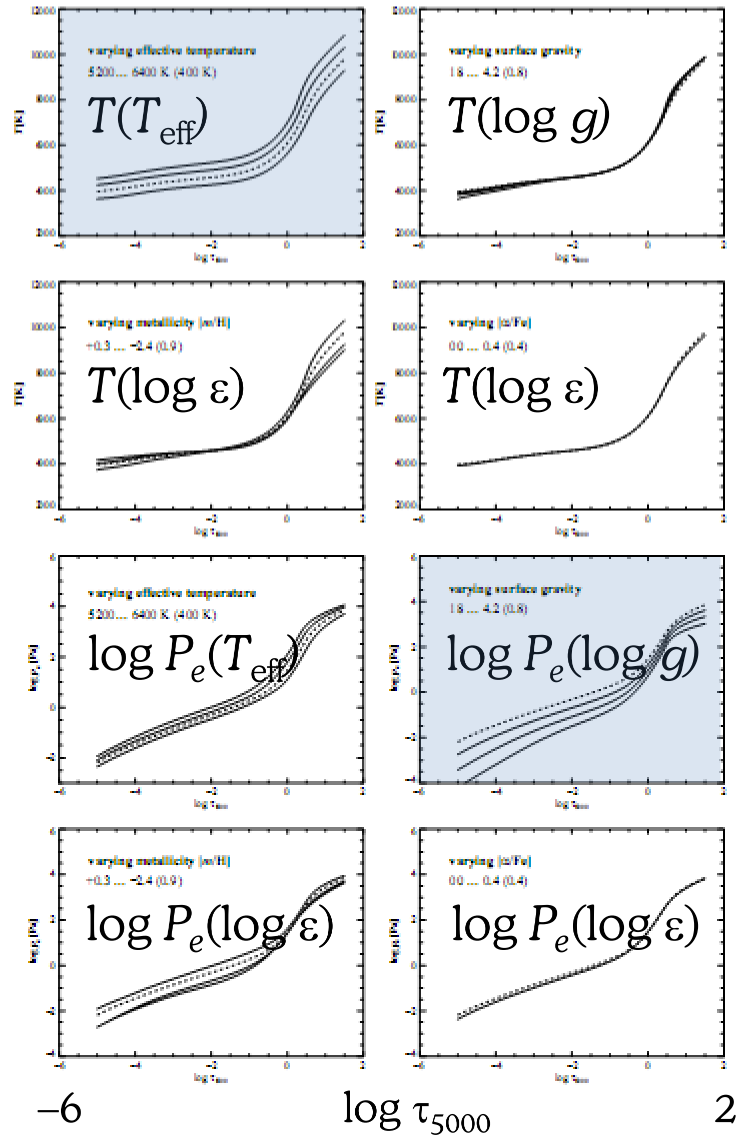
log $\varepsilon(X_i)$ (chemical composition)

hydrostatic equilibrium

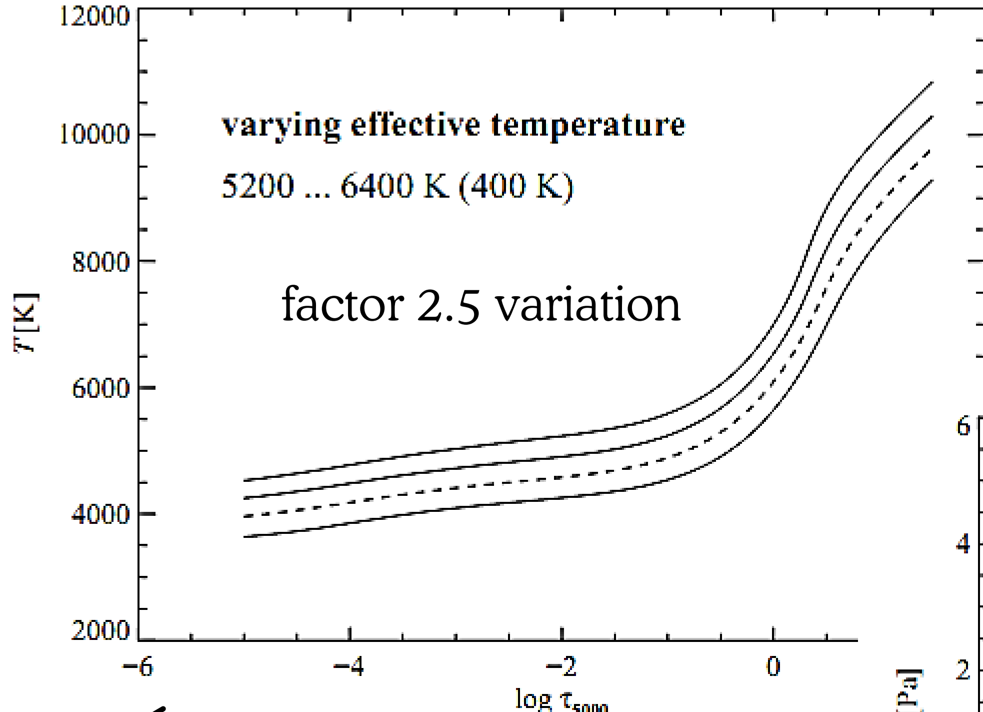
LTE (local thermodynamic equilibrium)

MLT (mixing-length theory) and

a statistical representation of opacities (either via opacity distribution functions, ODF, or opacity sampling, OS).

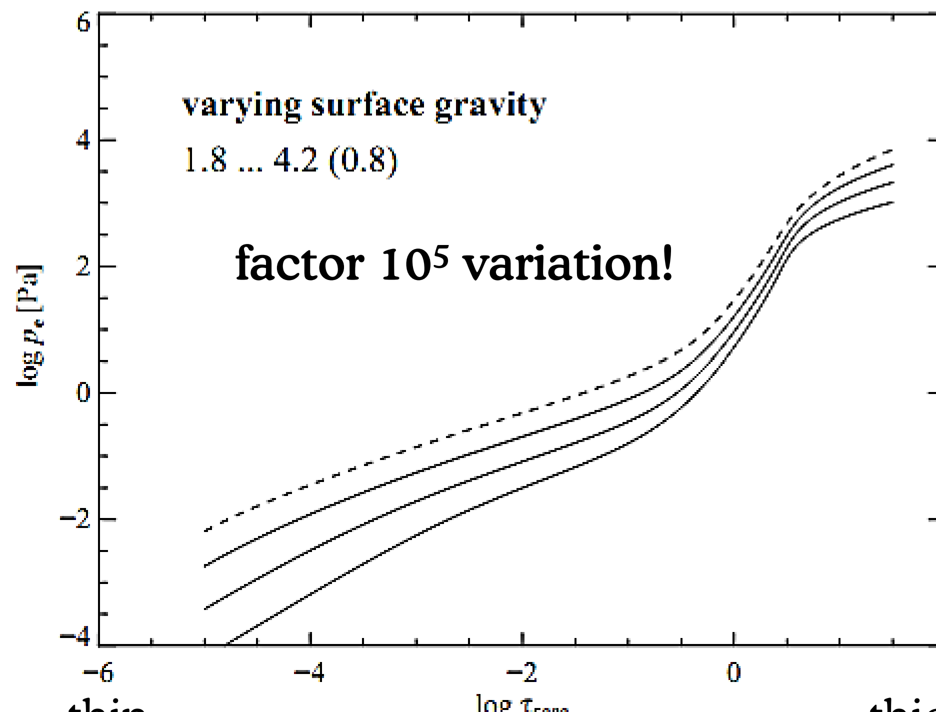


T vs *P* variation



towards interior

towards surface



thin

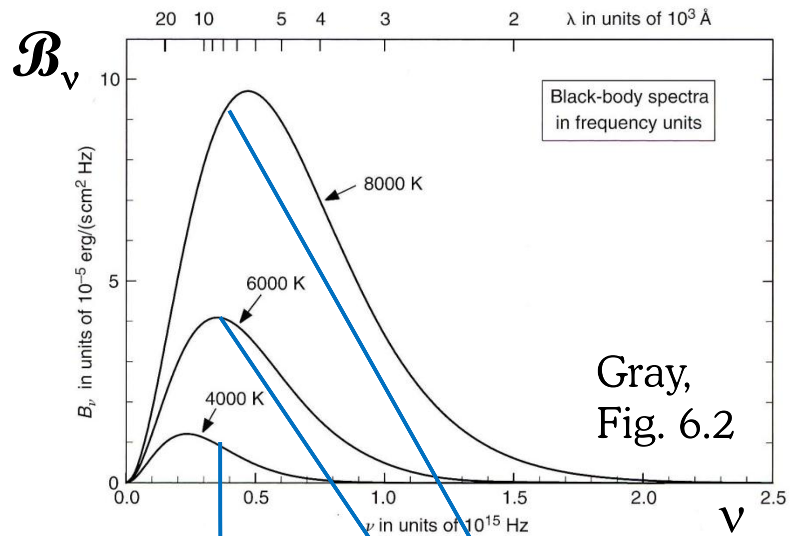
thick

(disgegarding the temperature rerise, the chromosphere, the corona)

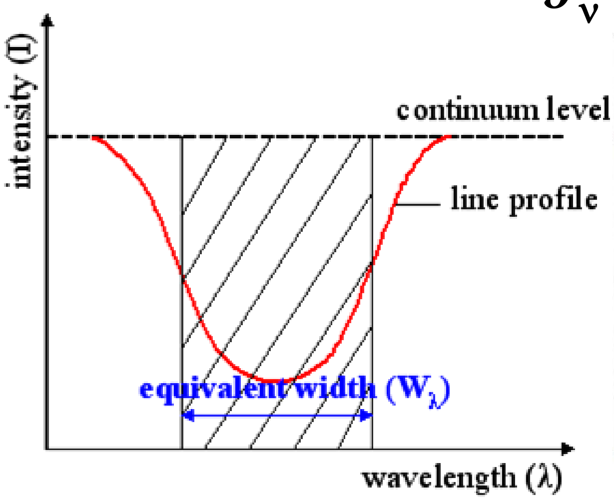
How spectral lines originate

The formation of absorption lines can be qualitatively understood by studying how
$\mathcal{S}_\nu$ changes with depth.

$$W_\lambda \propto d \ln \mathcal{S}_\nu / d\tau_\nu$$



Gray, Fig. 6.2



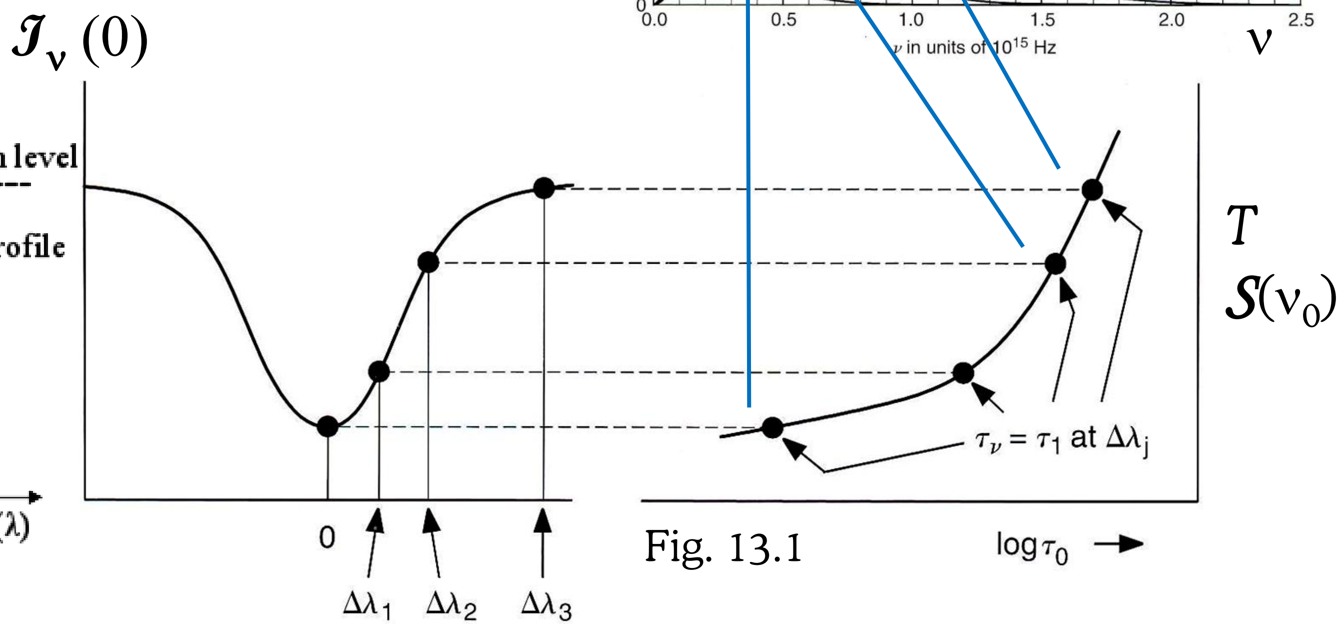


Fig. 13.1

Spectral lines as a function of abundance

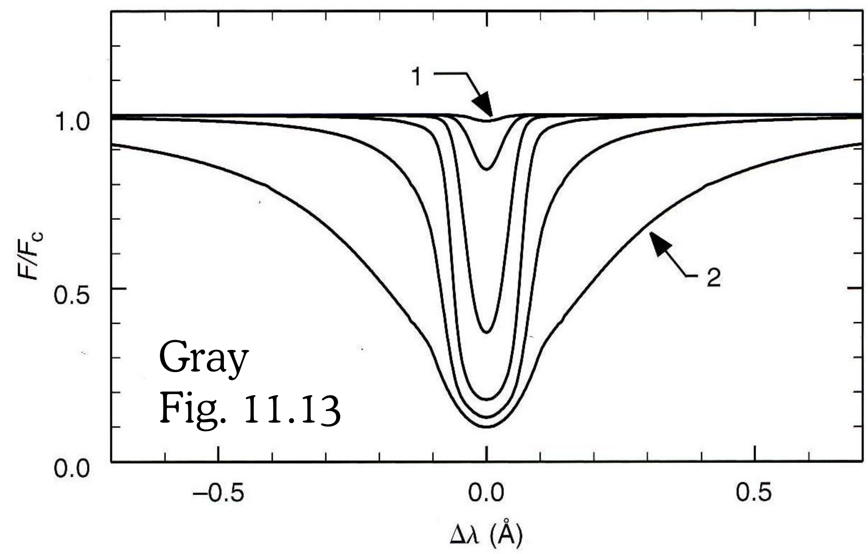
Starting from low log ε (low log *gf*), the line strength is directly proportional to log *gf*ε:

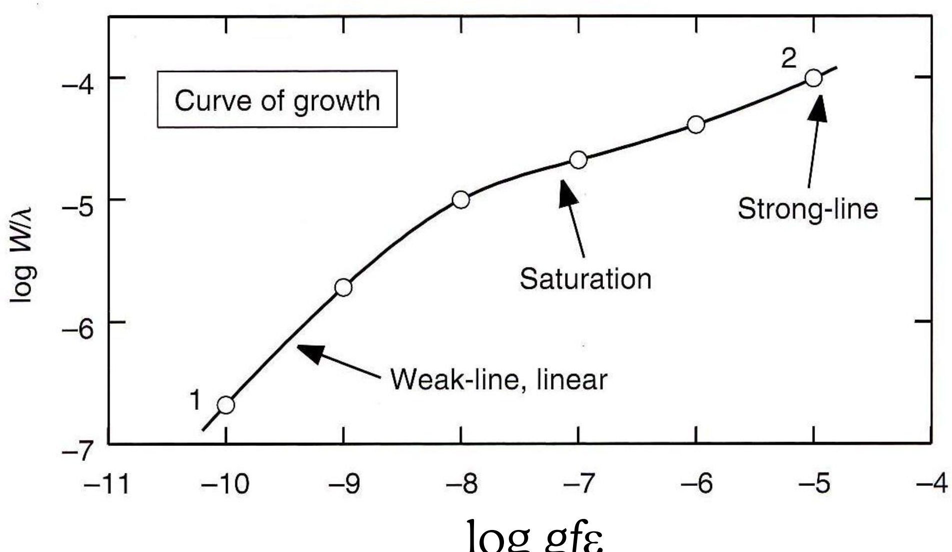
$$W_\lambda \propto gf\, n_X$$

When the line centre becomes optically thick, the line begins to saturate. The dependence on abundance lessens. Only when damping wings develop, the line can grow again in a more rapid fashion:

$$W_\lambda \propto \mathrm{sqrt}(gf\, n_X)$$

Weak lines are thus best suited to derive the elemental composition of a star, given that they are well-observed (blending!)





Broadening of spectral lines

There are numerous broadening mechanisms which influence the strength and apparent shape of spectral lines:

microscopic

1. natural broadening
(reflecting $\Delta E \, \Delta t \geq h/2\pi$)
2. thermal broadening
3. microturbulence ξ_{micro}
(treated like extra thermal br.)
(4. isotopic shift, *hfs*, Zeeman effect)
5. collisions (H: γ_6, log C_6; e^-: γ_4)
(important for strong lines)

macro

6. macroturbulence Ξ_{rt}
7. rotation
(8. instrumental broadening)

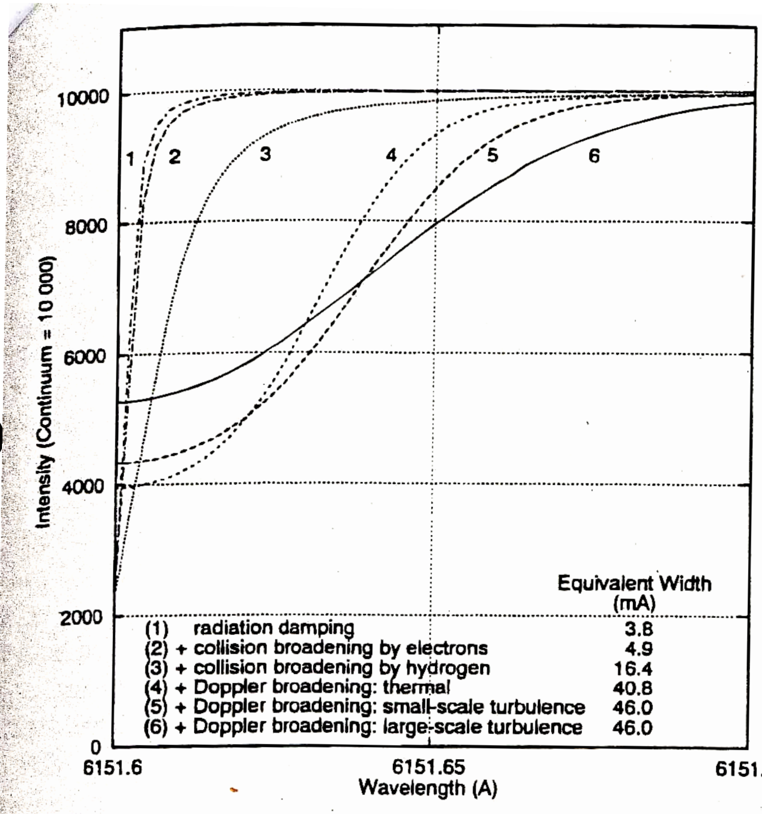


Fig. 3. Synthetic (half-)profiles of Fe I 6151.6 Å (Mult. 62, E.P. = 2.2 eV) showing the cumulative effect of various broadening mechanisms.

Holweger (1996)

Microturbulence and damping

If lines of intermediate or high strength return too high abundances, then the microturbulence or the damping constants are (both) underestimated (the *gf* values can also be systematically off).

Use an element with lines of all strengths to determine ξ. In most cases, this will be an iron-group elements.

Hydrodynamic ("3D") models are presently in an adolescent phase and will hopefully do away with the need for micro/macro-turbulence.

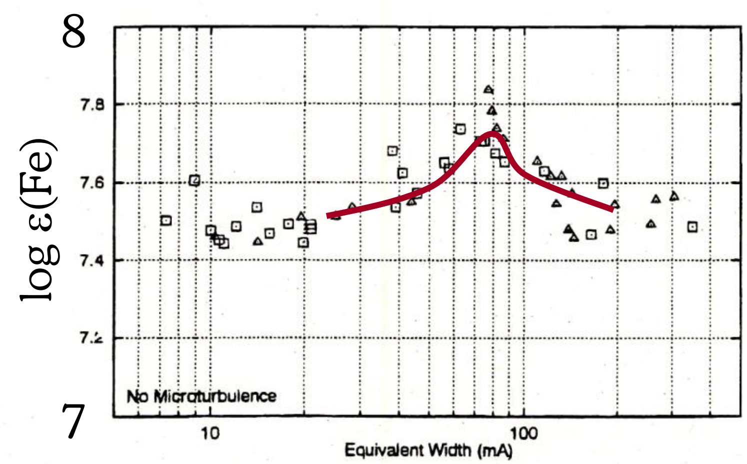


Fig. 7. Same as Fig. 6, but neglecting microturbulence.

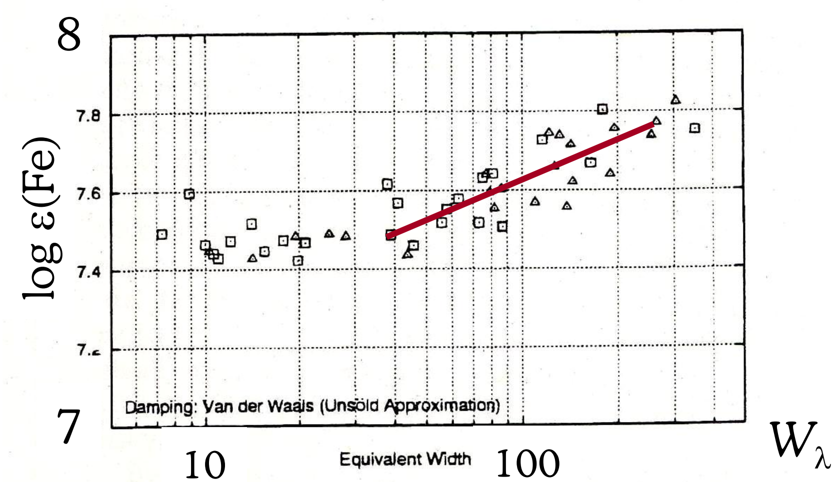


Fig. 5. Iron abundances derived from individual solar Fe I lines and Hannover *gf*-values. The two samples shown are from [4] (squares) and [18] (triangles). The deviation of the stronger lines indicates that the adopted damping constants are too small.

Broadening of spectral lines: an example

The Ca II triplet lines are broadened by elastic collisions with hydrogen:

$$\mathrm{Ca} + \mathrm{H} \rightarrow \mathrm{Ca}^* + \mathrm{H}^*$$

Detuning $\Delta\nu = C_n / R^n$: here C_6

Progress in the QM description of this interaction has led to a better understand of the profiles of these (and many other) lines (Anstee & O'Mara 1991, 1995).